ANALYTICAL MODELING OF AN ON-ROAD IN-PIT HEAVY TRAFFIC SITUATION

ANALITYCZNE MODELOWANIE SYTUACJI ZATŁOCZENIA WYWROTEK NA DRODZE W KOPALNI STOŻKOWEJ

Jacek M. Czaplicki - Mining Mechanization Institute, Silesian University of Technology, Gliwice

There are a few sensitive points in the operation of shovel-truck systems where a heavy traffic situation can be observed, e.g. power shovels should work almost round-the-clock and for this reason haulers should nearly always be available for loading. However, there are some other places where heavy traffic is distinctly undesirable – a queue of failed trucks waiting for repair should be almost negligible and moving trucks should have free untroubled movement on mine roads, especially where loaded units are concerned. In the paper the discussion is focused on the analytical modelling of an on-road in-pit heavy traffic situation that can occur when the number of circulating trucks in the pit is large. Three methods of modelling such a situation are presented and indications are given as to what kind of measures can show that a mutual truck disturbance might occur. The first method is based on a reliability theory model. The second proposition makes use of the modelling and analysis procedure given in a monograph (2009) by the author of this paper. The third method applies a model taken from queue theory. An additional topic that is examined is how to reduce this kind of heavy traffic situation by changing the organization of the truck system slightly.

W procesie eksploatacji systemów koparki-wywrotki pojawia się czasami sytuacja dużego zatłoczenia jednostek transportowych i to w różnych miejscach. Na przykład, koparki powinny pracować „na okrągło" i z tego względu wywrotki powinny czekać przy koparkach na załadunek tworząc kolejki. Bywają także takie miejsca, gdzie kolejki wywrotek nie powinny się pojawiać. Na przykład, kolejka uszkodzonych wywrotek oczekujących na naprawę przed warsztatem naprawczym. Innym miejscem, gdzie może pojawić się kolejka, to główna pochylnia wyjazdowa z kopalni, gdzie pełne wozy transportowe wywożą urobek na powierzchnię.

W pracy rozważania dotyczą tego ostatniego przypadku, gdy liczba wywrotek jest tak duża, iż jednostki te zaczynają przeszkadzać sobie wzajemnie i wydajność systemu zaczyna spadać. Trzy metody analizy takiej sytuacji są przedstawione. Jedna metoda jest oparta o model zaczerpnięty z teorii niezawodności. Druga metoda polega na wykorzystaniu pewnych informacji wziętych z ogólnej metody analizy i obliczania systemów koparki-wywrotki opisanej w monografii autora z 2009 r. Trzecia metoda bazuje na modelu G/G/k/r teorii obsługi masowej. W pracy zawarta jest także sugestia, jakie podjąć decyzje, aby zredukować zjawisko zatłoczenia wywrotek i aby zadanie transportowe systemu było możliwe do realizacji.

Introduction

The exploitation process of a shovel–truck system that operates in a given pit is very complicated by nature. This was proved in Czaplicki's monograph of 2009. Many basic and specific issues were discussed in the book cited but, as was stated at the end of monograph, 'much can still be done to improve the presented procedure'. This is true.

A certain subtle problem was omitted. That is an on-road in-pit heavy traffic situation that can occur during the operation of a shovel-truck system.

Usually, when the term 'heavy traffic situation' is mentioned two associations come into the minds of engineers. One association concerns the common everyday driver's life that happens on roads in many countries. This association is obviously quite a practical one and is a nuisance by nature. The second association may be related to theory, the theory of queues and it concerns a situation where a certain service system is almost always busy because of a nearly never-ending queue of clients waiting for service. Both associations are obviously related to each other but the latter one is connected with the mathematical feature of such situation. It is an extreme state where mathematical formulas are necessary for determining the features of this situation to illustrate terminal properties; some functions tend toward their limits. If we now look at the operation of a shovel-truck system operating in a pit, both associations have their specific sense.

In the monograph cited a 'heavy traffic situation' that can exist in the operation of a machinery system was discussed but not exhaustively. It was the one condition that should be satisfied in order to achieve a good statistical evaluation of the system. A lack of this constraint, in turn, was extremely necessary when the operation of the repair shop was analyzed. However, this limitation was an appropriate one to have the conviction that a shovel-truck system operates in the way it should. Explaining it in a more precise way, operating shovels (here: *service points*) should almost always be busy. This means that a lot of haulers should circulate in pit. And that is all. But this kind of heavy traffic situation has an influence on what goes on in the pit.

The goal of this paper is to present considerations on the influence of a heavy traffic situation that can appear on pit roads when many trucks operate there. This kind of heavy traffic is different in its impact on the machinery system considered than that usually taken into account in the analysis of the operation of shovel-truck systems. According to the best knowledge of author of this paper the problem considered here has no literature besides Czaplicki's and Kulczycka paper [4] published in 2011.

A phenomenon considered here has strict connection to so-called bunching well known in theory of traffic.

Factors having influence on a state of heavy traffic appearance

Let us assume that there is an open pit mine and a system of shovels operating in it. The number of these loading machines was calculated following the production requirements of the mine. Then a calculation of a truck system was performed resulting in a determination of the number of haulers that will be directed to the pit to haul the blasted rock given by power shovels. Obviously, the number of trucks in reserve was also estimated and the number of so-called 'surplus' units was assessed as well [1]. In such a way the truck system was fully specified in terms of its size and organization.

Introduce now the following notations: n is the number of power shovels, m is the number of trucks directed to the pit to execute hauling tasks and *r* is the reserve size.

Let us presume that the number of loading machines is not so great and for this reason the circulating truck fleet is not big. The truck system operation runs as smoothly as could be expected. Let us now assume that we start to increase the number of loading machines following an increase in the number of drilling machines applied due to certain reasons. This means that the mass of broken rock that should be removed from the pit increases causing an increment in the transportation task given to the haulers. The number of trucks should be increased in order to keep the loading machines busy. Therefore, the number of circulating machines in the pit rises. At a certain moment in this enlargement process the state in the pit becomes a *heavy traffic situation*. There are many moving machines on the mine roads and they start to disturb each other. The average truck speed slows down and the efficiency of the truck system diminishes. The moment of the occurrence of a state of heavy traffic is a subtle and stochastic one. As the number of trucks grows slowly some drivers – during their journeys – face a situation from time to time when they have to slow down their truck because of moving units close to them. Sometimes they must even halt the truck for a moment. The number of such events will grow alongside the increase in the number of moving units.

The occurrence of these events mainly depends on:

(a) the number of trucks moving on mine roads,
(b) the velocity of the moving haulers,
(c) the skill of the drivers,
(d) the configuration of the roads in the mine,
(e) the quality of these roads (especially road width and the quality of the road surface),
(f) the selected technical parameters of the trucks (total mass of truck, length of braking distance for a given speed of a truck etc), and
(g) the truck dispatching system implemented in a mine.

Let us briefly discuss the above-listed factors.

The first factor (a) is obvious. The more vehicles in motion in a pit, the greater the possibility of the occurrence of a heavy traffic situation.

If trucks are moving slowly (b) there is more time to properly react to any situation that might occur on a mine road; therefore fewer situations to disturb other trucks.

A driver with more experience reacts more correctly (c) – again, there are fewer situations to disturb other trucks.

If the mine road has a lower gradient, trucks move more quickly. If there are many sharp bends on the road, the speed is lower. In addition, some types of configurations of mine transporting roads (d) are more convenient for keeping trucks in circulation without disturbances (e.g. torrent organization – one inlet and a separate outlet, i.e. the hauler goes all the way through; antonym: loop type).

It is obvious that if the road width is wider and/or the quality of the road surface is better (e) there is a safer and smoother movement of trucks.

The greater the total mass of a truck and/or the higher the vehicle speed, the longer the braking distance (f).

Some dispatching rules promote a crowded situation in a pit (h), i.e. rules that prefer more trucks in circulation.

Heavy traffic measure

In spite of the fact that the number of factors having an influence on the frequency of the occurrence of these events is high, a decisive factor – as a rule – is this first one. The basic measure in this regard is the density package, i.e. the number of trucks moving on the mine roads per unit length of the mine roads on which these moving units operate. However, this statement is correct only at first glance; problems arise when one starts to consider this measure more precisely.

The number of trucks in motion on mine roads at a given moment is a random variable of unknown probability distribution. What can one say about it? A probability distribution that is known is the probability distribution of the number of trucks in a work state determined based on the Maryanovitch model provided that the repair shop is arranged correctly (Czaplicki 2009 [2] Chapter 7.2). Recall that for a number of trucks equalling m, the mass of probability is significantly greater than for other values. This is due to the existence of spare haulers in the system.

Now, the question "where are these 'good' vehicles?" arises. The majority of them are in motion, empty or fully loaded. But, a certain number of trucks are waiting at shovels for loading and a certain number of haulers are being loaded. Perhaps, a few of them are dumping their loads. Those trucks at shovels and unloading points do not generate a heavy traffic situation on roads; they are neutral. Thus, we have to 'remove' these neutral units from further considerations. The proper way to estimate the probability distribution of interest is to identify the probability distribution of the number of neutral trucks and subtract the random variable of the number of neutral trucks from the random variable of the number of trucks in a work

[1] For a system of this kind there will be a certain number of trucks – on average – that will not be available for hauling and these units will not be in the reserve. These haulers that sit with their engines off because of refuelling, changing drivers, planned maintenance etc. are called 'surplus units'.

state. Unfortunately, this way is very complicated to evaluate, especially taking into account the fact that these two random variables are stochastically dependant. Even when we write it down in mathematical formulas their practical estimate will be very difficult to obtain. Therefore, a different approach is needed.

Before different measures for this kind of heavy traffic situation in diverse areas is constructed, let us make a more precise analysis of what is happening on the mine roads.

Looking more carefully at the pit transporting routes one can easily come to the conclusion that segments of the transporting roads are not equal to each other in terms of the intensity of their usage. It is easy to see that:

(a) The main access ramp is the most important
(b) Of less importance are the segments of roads on the production levels
(c) Of low importance are those final segments which are only connected with loading machines.

Because the intensity of truck movement in case (a) is clearly higher that in the other cases we will devote our attention to the traffic on the main ramp.

Presume that it is a two-lane ramp. One stream of trucks fully loaded creeps up; the second stream flows down comparatively quickly because of their empty boxes. Light vehicles have greater manoeuvrability, are relatively easy to stop and their speed ensures their quick arrival at their destination – all of these compared to heavy units. Hence, the problem of the disturbance of truck movement is usually more serious in the stream of heavy units. For obvious reasons the number of units driving down is lower than the number of haulers driving up at any given moment. Therefore, the main point of interest for a heavy traffic situation due to a disturbance of truck movement is the main hauling ramp and haulers going up.

Let us go into this consideration a little further. In a general case the loading machines operating in a pit are not located at the same level. Therefore, trucks enter the main ramp at different points. This means that the main incline is used with a different intensity if its length is considered. Following this line of reasoning it is easy to come to the conclusion that the most important part of the main ramp is determined by the first i.e. shallowest production level and the ramp outlet on the bank. And this segment should be used in any further analysis. Denote this part by *SMIU* (segment most intensively used).

First approach – reliability one

We can analyze a crowded situation on an *SMIU* by applying different models. Presume that the point of interest is an inlet of the *SMIU*. One can treat the arrival process of trucks on the *SMIU* as the point process in which 'a point' is the moment of the appearance of a hauler on an inlet. Such a process is well-known in reliability theory and is called the *stream of failures model* where failure here is equivalent to the appearance of a truck on the inlet.

A process of this kind has been investigated and comprehensively described by Smith (1958, 1959) [7, 8], Gnyedenko et al. (1965) [5] and others [2]. What kind of measures should be of interest for our case?

There is no doubt that the renewal function *H(t)* which determines the expected number *n(t)* of failures in time *(0, t)* is important; here: the number of trucks occurring at the inlet of the *SMIU* up to the moment *t*. It is defined by the equation:

$$H(t)=\sum_{n=1}^{\infty}F_n(t) \tag{1}$$

where:

$$F_n(t)=\int_0^t F_{n-1}(t-t_w)dF(t_w) \qquad F_1(t)=F(t) \tag{2}$$

$$P\{n(t)>n\} = P\{t_n<t\} = P\{t_{p1}+ t_{p2}+t_{p3}+\ldots+t_{pn}\} = Fn(t) \tag{3}$$

t_{p1}, t_{p2}, ... are succeeding moments of the appearance of a truck at the *SMIU* and *F(t)* is the probability distribution. It is assumed that these random variables have identical probability distributions.

Denote by T_R the recommended distance between two neighbouring moving trucks formulated by the truck producer and accepted by mine management. A point of interest should be the value of the function *H(t)* at point T_R. It should fulfil the inequality

$$H(TR) < 1, \tag{4}$$

otherwise there is a heavy traffic situation for sure.

Now relate these relationships with mine practice. It can be assumed that the probability distribution of the times between two neighbouring arrivals at the inlet of the *SMIU* is normal and for this reason

$$H(T_R)=\sum_{n=1}^{\infty}\Phi\left(\frac{T_R-nT_0}{\sigma\sqrt{n}}\right) \tag{5}$$

where

$$\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left(-\frac{x^2}{2}\right)dx \tag{6}$$

provided that $\sigma << T_0$ and T_0, σ denote the average time between two neighbouring arrivals at the inlet of the *SMIU* and the corresponding standard deviation.

It is worth adding that the density of the arrivals is given by the pattern

$$h(t)=H'(t)=\sum_{n=1}^{\infty}\frac{1}{\sigma\sqrt{2\pi n}}\exp\left(-\frac{(t-nT_0)^2}{2n\sigma^2}\right) \tag{7}$$

and has the characteristic shape of suppressed oscillation tending to T_0^{-1} level when *t* grows Fig. 1. Actually, this should not be a problem in estimating function (5).

Summing up – making use of this type of investigation one takes into consideration only the inlet of the *SMIU* of the main

[2] It was also recalled by Czaplicki in 2010 [3].

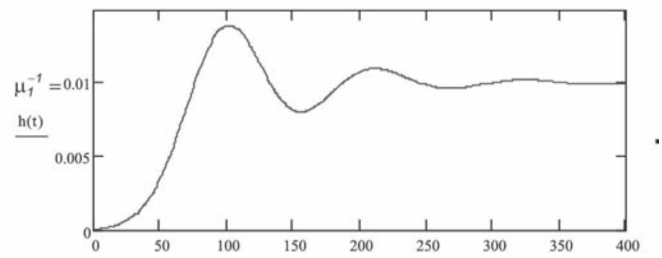

Fig. 1. Probability density function *h(t)* of number of trucks arriving at *SMIU*

Rys. 1. Funkcja gęstości *h(t)* przybyć wywrotek na odcinek pochylni o największym natężeniu ruchu

transporting ramp. Notice that the all properties of the exploitation process of the machinery system analyzed are hidden in the properties of the stream of trucks arriving at the inlet of the *SMIU*.

An approach based on the procedure of an analysis of shovel-truck systems (Czaplicki 2009 [2])

The procedure of modelling and comprehensive analysis of the operation of shovel-truck systems given in the monograph cited allows the key problem discussed here to be examined. Two parameters are needed in order commence considerations, namely:

- E_{wkd} – the conditional expected number of trucks at power shovels for d shovels able to load; $d = 1, 2, ..., n$
- $P_{kd}^{(zd)}$ – the probability that d shovels are able to load and (n-d) spare loaders are available to load.

The unconditional expected number of trucks at the loading system can be calculated using the formula:

$$E_{wlk} = \sum_{d=1}^{n} E_{wkd} P_{kd}^{(zd)} \quad \text{trucks.} \tag{8}$$

Having this result in mind one can presume that the number N_{mt} of trucks moving on the mine roads is given by the approximate equation

$$N_{mt} \approx m - E_{wlk} \quad \text{trucks.} \tag{9}$$

This number can be corrected down by assuming that a few haulers may be at the dumping points.

Let now assume that we have an idea of the approximate percentage g_p of the number of trucks driving up on the *SMIU* in relation to the total number of moving haulers. The number of trucks moving on this segment is

$$N_r = \frac{g_p}{100} N_{mt} \quad \text{trucks.} \tag{10}$$

Calculate the density package d_p of moving trucks on the *SMIU* in units of ramp length L_r

$$d_p = \frac{g_p}{100} N_{mt} \frac{1}{L_r} \quad \text{trucks/unit of length.} \tag{11}$$

Now, we can construct a measure which allows an on-road in-pit heavy traffic situation to be verified. The following relationship should be confirmed

$$d_p T_R < 1 \tag{12}$$

otherwise, an on-road heavy traffic situation will reduce the performance of the machinery.

This result is only an approximate one.

Two significant observations

Our actual considerations should be enlarged by two important remarks.

(1) Considering attentively the procedure of shovel-truck system analysis, it is easy to see that for a given power shovel system and having information on the mean time of truck travel (haul-dump-return) plus the mean time of truck loading, the package density can be roughly determined. Information on the mean distance travelled by a truck is hidden in the information on the average time of truck travel. Thus, presuming that each loading machine should almost always be kept busy, the number of haulers strongly depends on the travel distance.

(2) Keeping the above in mind, one can find that there is no point in having a large number of power shovels in a pit because these machines need a lot of trucks to load and for this raison d'être there is a significant probability of the occurrence of a heavy traffic situation with a disturbance of truck movement. This is an operational constraint in the size of a power shovel system; other constraints can be economic and/or technical ones.

A queue theory approach

Here is the third possibility to trace a heavy traffic situation of the type being considered. It is based on the model from queue theory.

One can treat the *SMIU* as a kind of service system. Its scheme can be illustrated as is presented in Fig. 2. In this Figure the fact that the truck system is a closed and cyclic one is clearly marked. The properties of the system presented in Fig. 2 are as follows.

The number of circulating trucks is a random of the probability distribution determined by the Maryanovitch model that corresponds with the organization of the truck system. Hence, the reliability of the haulers is taken into account as well as the size of the truck reserve. A truck work cycle is a two-phase one: travel on the most used part of the main ramp (*SMIU*) and the *rest*. The second phase consists of several stages: travel from the ramp to the dumping point – unloading, travel back to a power shovel – loading – travel from the power shovel to the *SMIU* of the ramp. It can be assumed that the time of truck travel on the *SMIU* is a random variable normally distributed. It can also be assumed that the time of the second phase is also normal; however, this distribution is a compound one due to the different distances travelled by haulers to and from different loading units. It is also necessary to include the idle time spent by the truck in a queue for loading. This time also has a random character but is not normal. Its density function has a rather exponential shape or a near exponential figure. These two random variables should be added to get the random variable characterizing the random variable of the second phase. It is difficult to express it in an explicit form to get the appropriate statistical estimate.

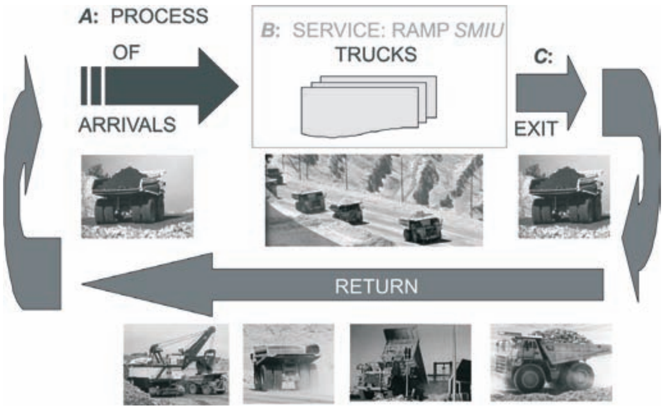

Fig. 2. An idea of the SMIU of main ramp as a kind of queue service system
Rys. 2. Idea potraktowania najbardziej obciążonego segmentu pochylni wyjazdowej jako elementu systemu obsługi masowej

The problem can be greatly simplified by applying the Sivazlian and Wang *G/G/k/r* model (1989) [6], but appropriately modified to the case being considered. Compared to the general model analyzed by the authors cited there is no reserve here [3] (simplification) and the number of circulating trucks is random (generalization) [4]. Obviously we are not considering the machine repair problem; therefore, the meanings of the phases are different. Simplification also relies on the fact that only two basic statistical parameters for each probability distribution are needed: the expected value and the corresponding standard deviation. The probability distributions can be any type. An operating scheme of a truck queuing system can be presented as is shown in Figure 3.

Truck system size versus truck circulation

Let us presume that there is a truck system and there are signals that too many units are in motion. In the truck dispatching room analytical indicators show that there is a tendency for truck output to decrease slightly. What should the rational decision be?

At first glance it seems obvious that the only solution is to withdraw some trucks from circulation and the problem will be sorted out completely. That is true, but not entirely.

The procedure of shovel-truck analysis given in the monograph cited allows one to think about a slightly different but better solution. We have to direct our attention to the stage of the procedure where the selection of the pair <*m, r*> is made. To commence this stage two parameters are required, namely the expected number of trucks in a work state *E*(*D*) and the number of trucks needed. Their calculation is quick and simple. Selection of the pair <*m, r*>, in turn, relies on searching for two numbers fulfilling two conditions (Czaplicki 2009 [2], Chapter 8):

(a) The total number of trucks applied in the system to accomplish the transportation task given should be minimum
(b) From all pairs fulfilling condition (a) the pair that has the maximum reserve must be looked for.

In the situation of on-road in-pit heavy traffic the above conditions should be rejected and a new criterion must be formulated:

We search for such a pair <*m, r*> that:
(1) The transportation task is accomplished
(2) The number of circulating trucks is minimal,

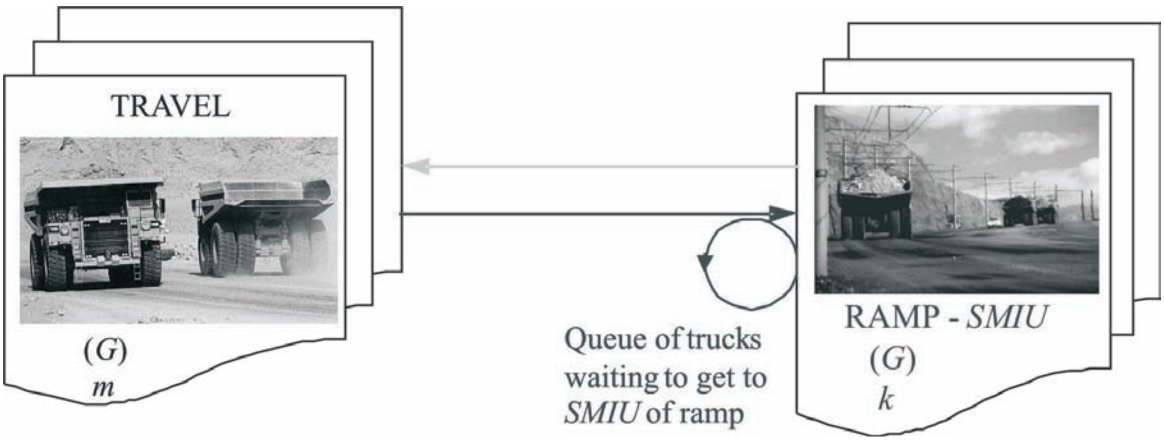

Fig. 3. Operating scheme of a truck queuing system with a ramp
Rys. 3. Schemat funkcjonowania systemu kolejkowego dla odcinka pochylni najbardziej obciążonego

Applying this model one obtains the probability distribution of the number of trucks that are theoretically on the *SMIU*. Obviously this part of the ramp has its own capacity bearing in mind the recommended safe distance T_R. If the number of haulers obtained from the model is greater than the ramp capacity, it means that some units are waiting to get to the incline; therefore, there is a heavy traffic situation on the ramp.

which means that in fact enlargement of the truck fleet is recommended, which initially sounds irrational [5].

However, the idea here is to have the minimum number of moving haulers and an enlarged reserve in order to almost always have the possibility to replace a failing truck by a unit from the reserve. Such a solution reduces the probability of the occurrence of a heavy traffic situation on the main transportation ramp.

[3] Information on the truck reserve is included in the Maryanovitch model.
[4] Examples of generalizations (the number of loading machines is random, the number of circulating trucks is random and the number of repair stands able to work is random) are comprehensively described in Czaplicki's monograph of 2009 [2].
[5] This problem was analyzed in Czaplicki's Polish monograph of 2006 [1], Chapter 5.5.

Nevertheless, the possibility of the application of such a solution is limited.

To illustrate this part of the analysis consider the following example.

There is a machinery system for which the expected number of trucks in a work state $E(D) = 55.9$ and the steady-state availability of trucks $A_t = 0.778$. Applying the Maryanovitch model and keeping in mind the original criterion after a short calculation one obtains the recommended pair *<m=61, r=11>*. Presume now that the truck dispatcher traced the problems with hauler movement because of too many trucks. If so, let us calculate all of the pairs for which the expected number of trucks in a work state is not lower than $E(D)$ and m is lower than *61*. Employing the Maryanovitch model again, after short calculations, we have:

<m=60, r=13> <m=59, r=14> <m=58, r=15>
<m=57, r=17> <m=56, r=23>.

Figure 4 is an illustration of outcomes obtained.

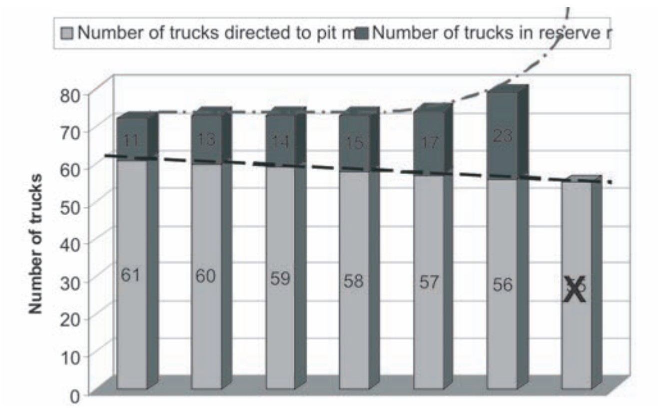

Fig. 4. Possible truck systems to accomplish the transportation tasks determined

Rys. 4. Możliwe systemy wywrotek dla realizacji określonego zadania transportowego

Let us make some brief comments. First the system is the original one. Further systems are characterized by lowering the number of trucks going to pit. Because of this reduction the reserve size grows; at first slowly, later rapidly. For only *56* units directed to the pit, *23* trucks are needed in the reserve to achieve the necessary production. In all cases the total number of trucks is greater than in the original system. For *55* haulers in operation, there is no way to reach the production target even if the reserve is infinite. Notice in the end that if the system is original and few haulers are withdrawn to the reserve, the transportation tasks will very likely not be achieved.

Enlargement of the truck fleet may be treated as exploring future mine needs in this regard.

Final remarks

Notice that the Sivazlian and Wang model was proposed twice here. It works if the condition of a heavy traffic situation is satisfied. Here only such a case is considered.

There is one subtle problem hidden in these considerations. It is the problem of the parameter of stochastic processes included in the analysis. At first glance, it looks as if there is no significant difference whether the process parameter is the time or the distance travelled. But this is not true.

A factor that makes the difference between these two parameters is the angle of inclination of the ramp. The relationship between them is obvious and is taken into account by truck dealers when providing information on the recommended distance between two moving units.

In all of these cases rich statistical data is needed. However, careful attention should be paid to possible changes in the size of the truck fleet when the data was gathered. Some process parameters change significantly when the number of haulers in the system increases. Data is not homogeneous. This fact must be taken into considerations; otherwise incorrect outcomes will be obtained from the analysis.

There is no doubt that this problem is interesting from both the theoretical and empirical points of view and further investigation seems to be desirable.

References

[1] Czaplicki J. M. *Modelling of exploitation process of shovel-truck system. Monograph.* Papers of Silesian University of Technology, 1740, Gliwice 2006. (In Polish)

[2] Czaplicki J. M. *Shovel-truck systems. Modelling, analysis and calculation.* CRC Press, Taylor & Francis Group 2009.

[3] Czaplicki J. M. *Mining equipment and systems. Theory and practice of exploitation and reliability.* CRC Press, Taylor & Francis Group 2010

[4] Czaplicki J. M. and Kulczycka A.: *Investigating an on-road in-pit heavy traffic situation. International Journal of Mining and Mineral Engineering.* Vol. 2, No 4, 2011, str. 338-348

[5] Б. В. Гнеденко, Ю. К. Беляев, А. Д. Соловьев, *Математические методы в теории надёжности.* Изд. Наука, Москва 1965
– Translation: B. V. Gnyedenko, Yu. K. Belayev and A. D. Solovyev, *Mathematical methods of reliability theory.* Academic Press. New York 1969

[6] Sivazlian B. D. and Wang K. H. *System characteristics and economic analysis of the G/G/R machine repair problem with warm standbys using diffusion approximation.* Microelectronics & Reliability, 29, 5, 1989, pp. 829-848

[7] Smith W. L. *Renewal theory and its ramifications.* Journal Roy. Soc. Ser. B, 20, 1958, pp. 243-302

[8] Smith W. L. *On the cumulates of renewal processes.* Biometrica, 46, 1959, pp. 1-29