Statistical characterization, probability distributions and spatial variability of lignite spoils

Alexandros Theocharis, Ioannis Zevgolis, Nikolaos Koukouzas, Christos Roumpos, Tryfon Mparbas

6-10-2022, Wroclow, Poland

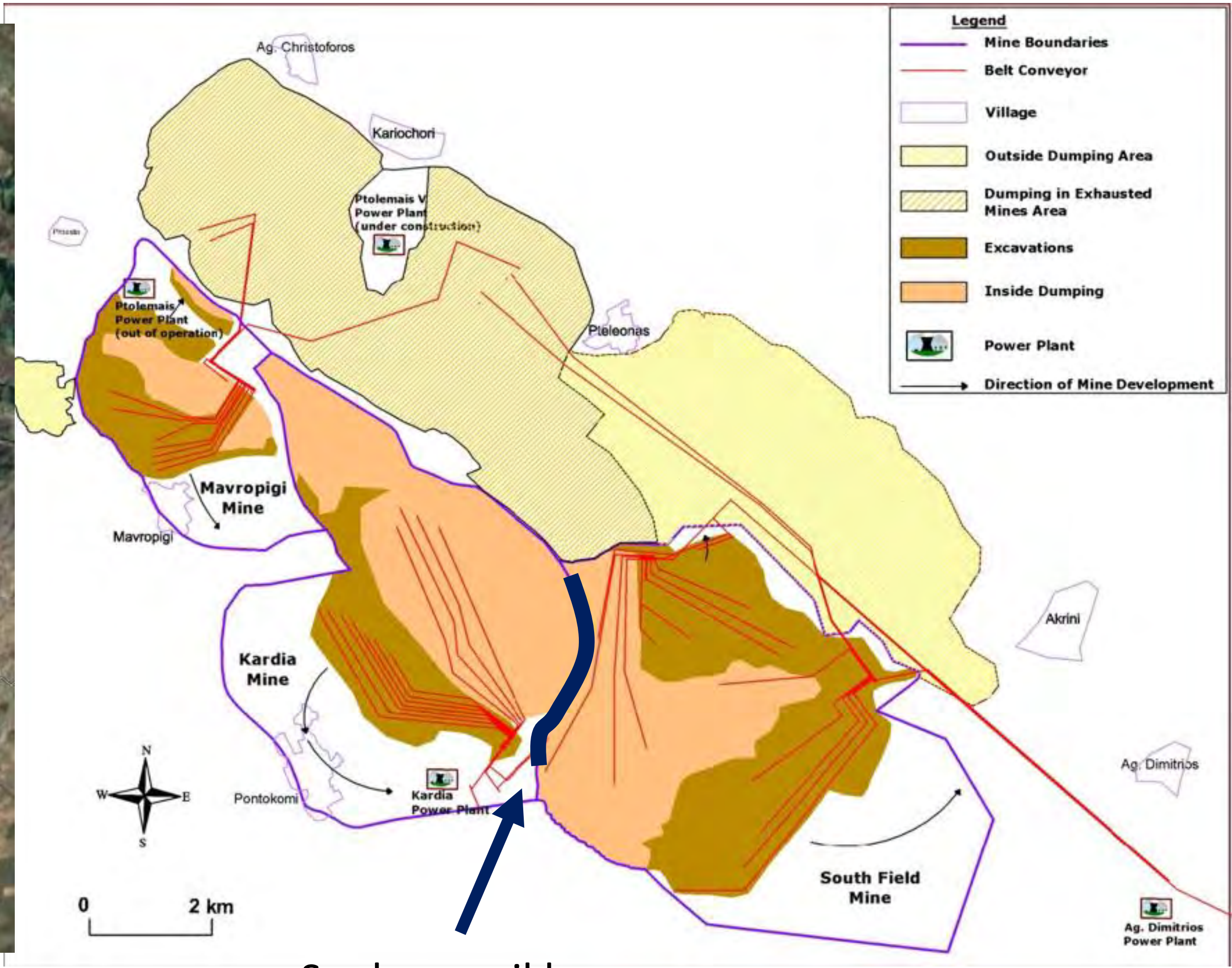
Table of contents

- Basic statistics
- Distributions
- Spatial variability

General overview of the Ptolemais mines and the Soulou spoil heap (mid 2018)

Soulou spoil heap

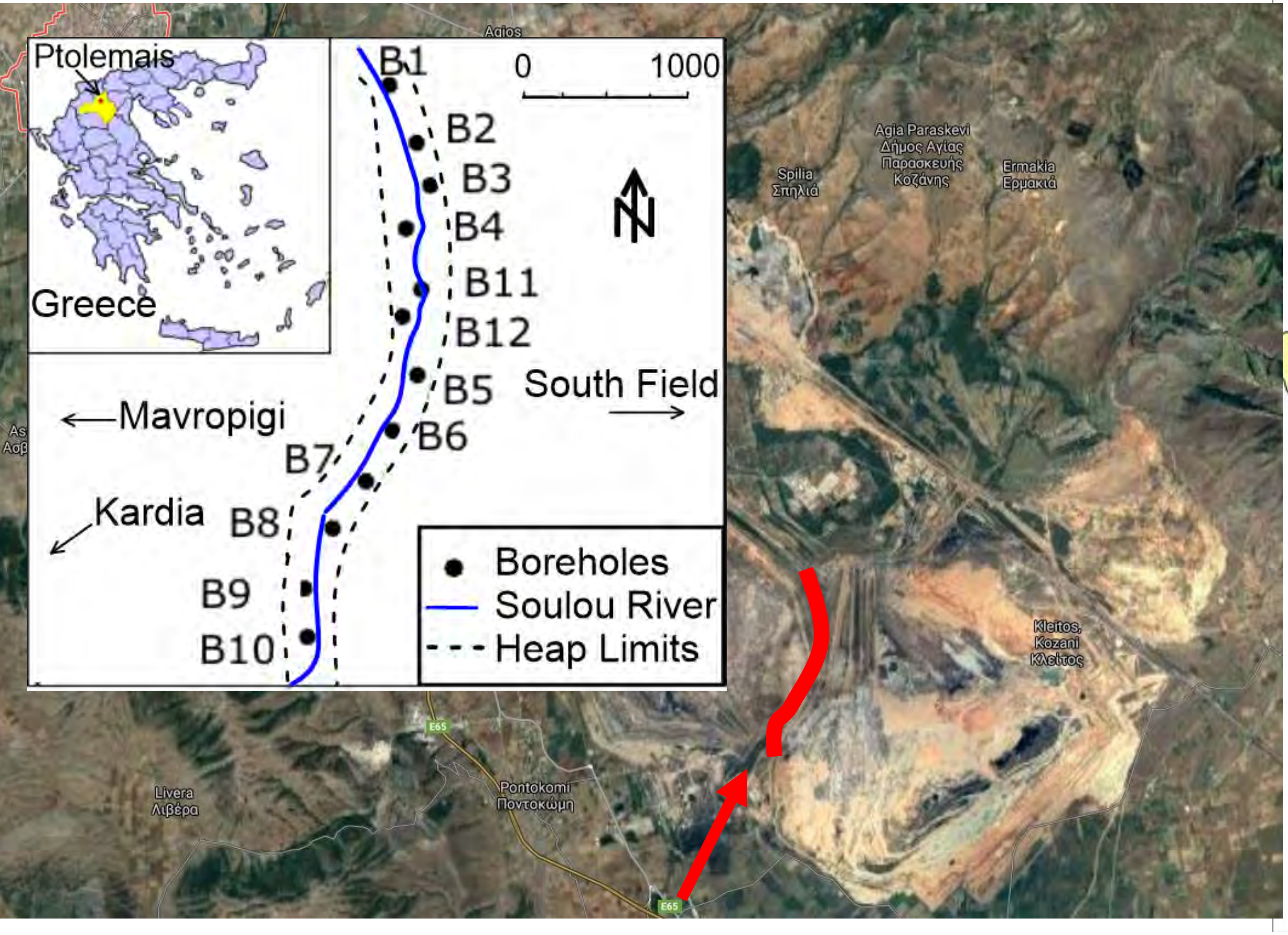
Google maps: satellite view of the mining area



Soulou spoil heap

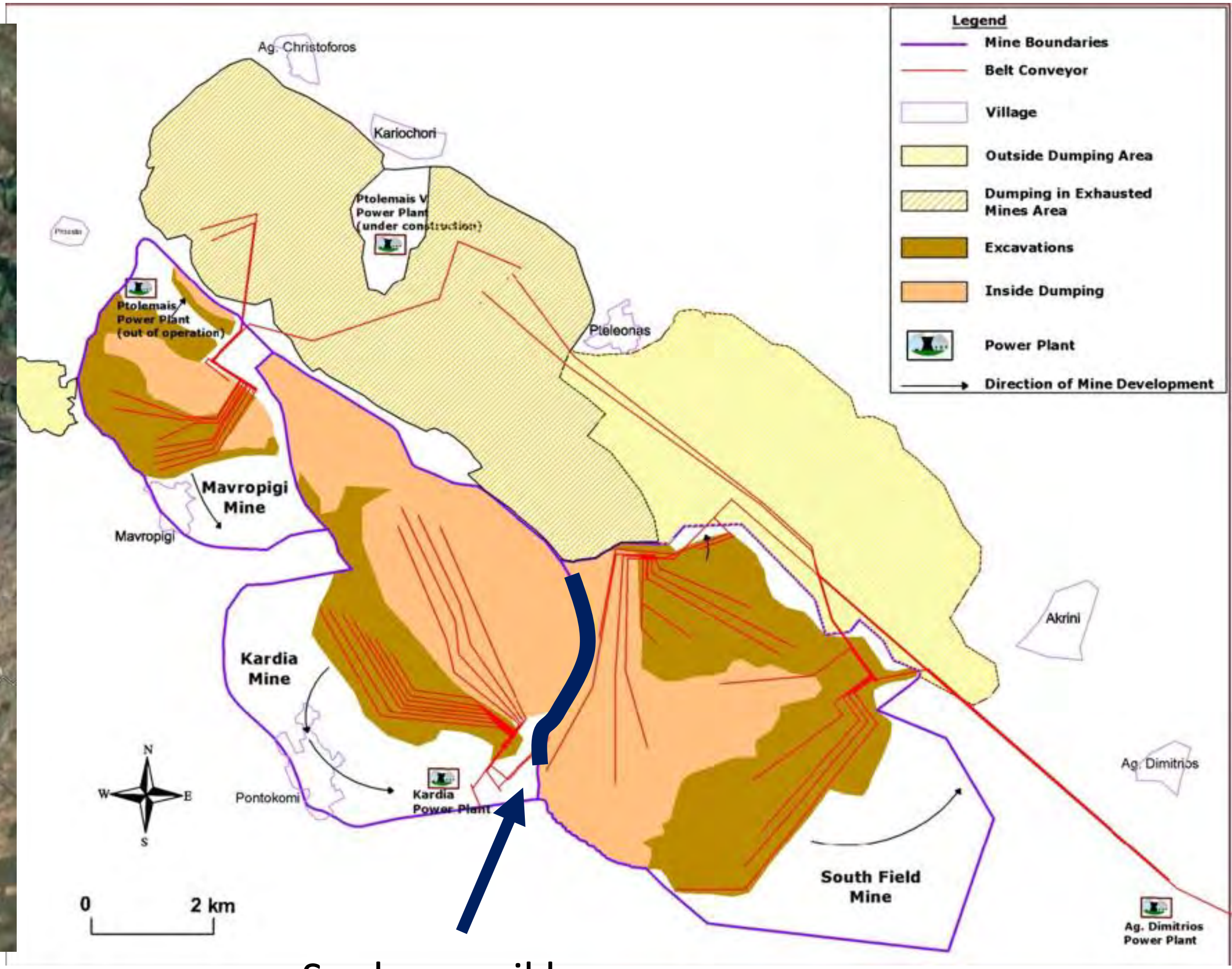
PPC 2020

General overview of the Ptolemais mines and the Soulou spoil heap (mid 2018)



Soulou spoil heap

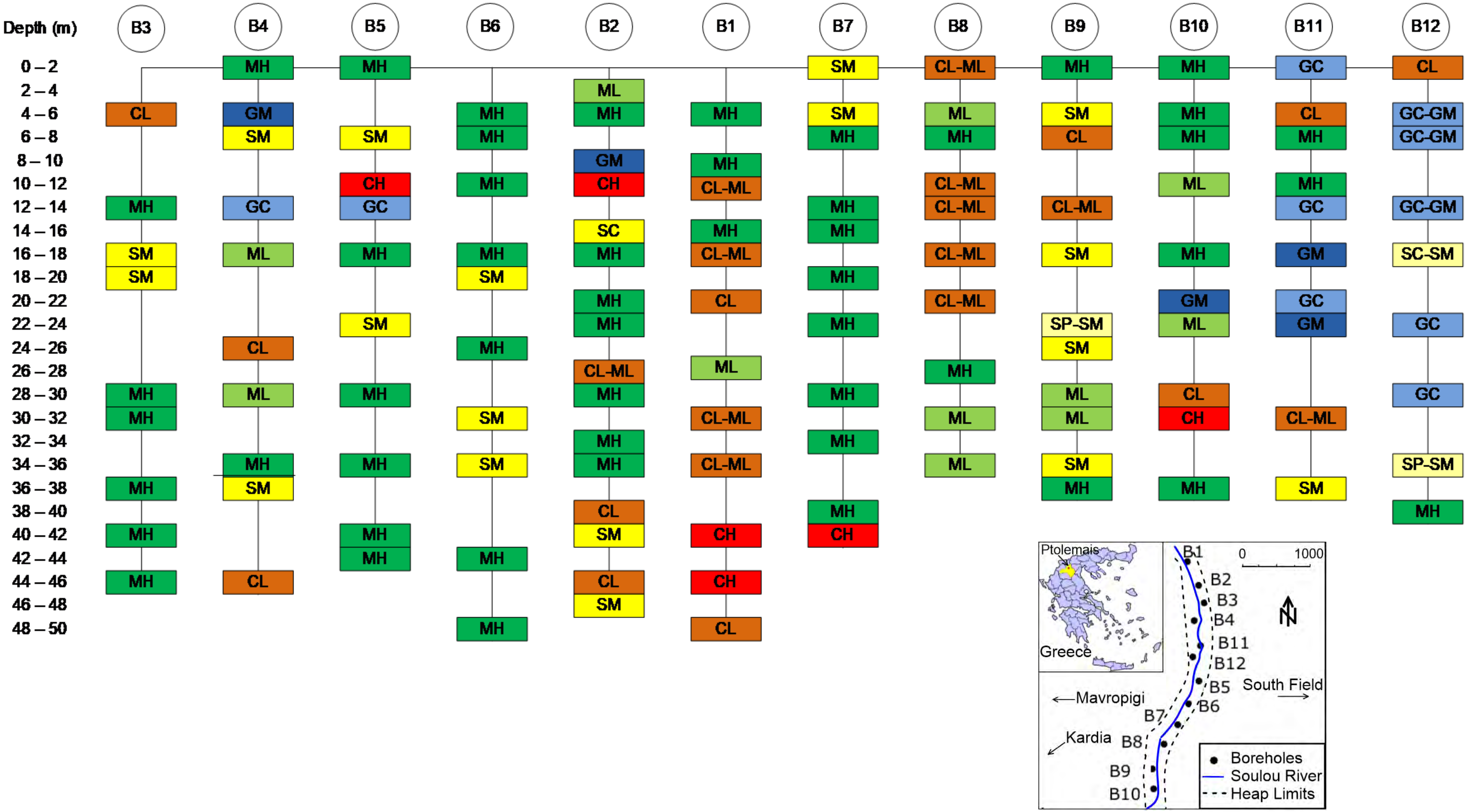
Google maps: satellite view of the mining area



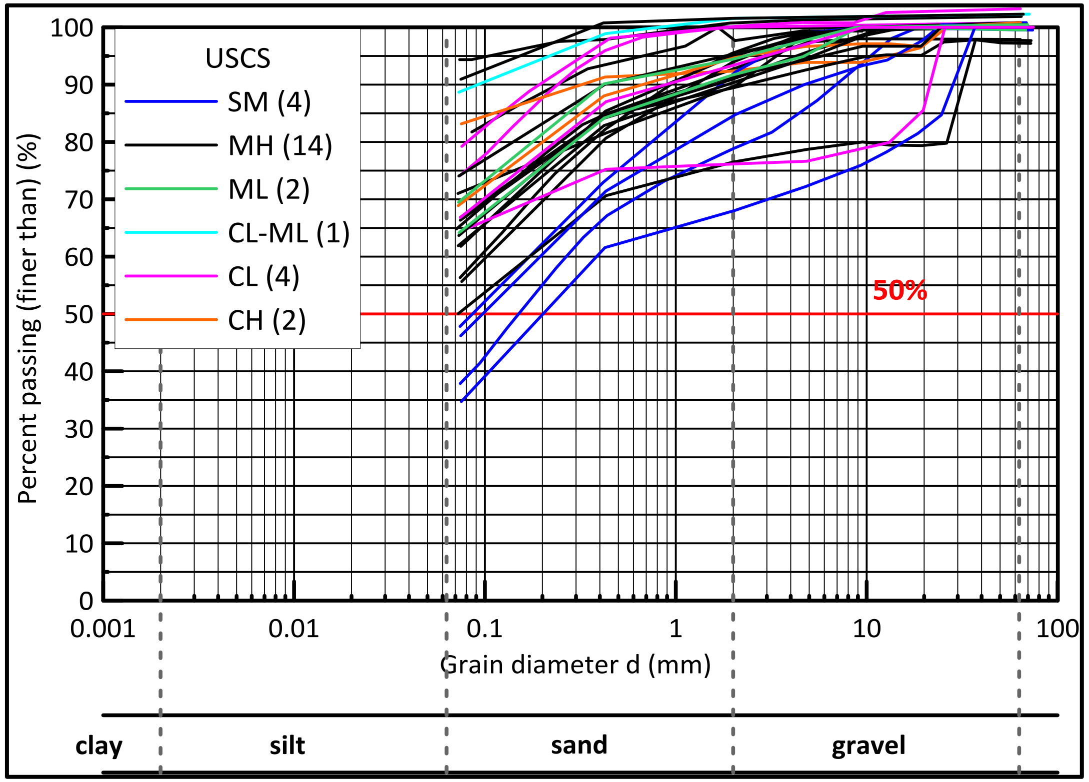
Soulou spoil heap

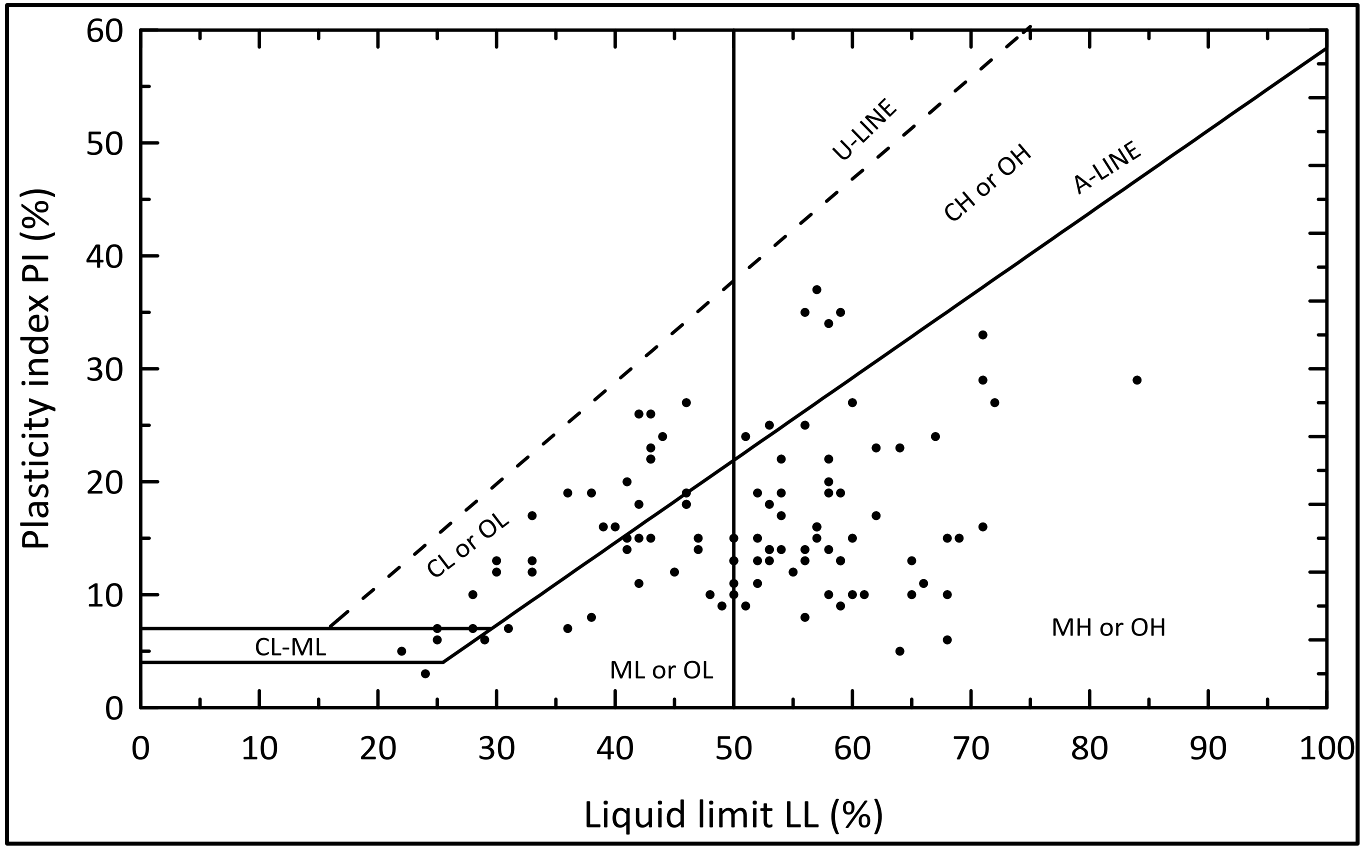
PPC 2020

The Soulou spoil heap - Geotechnical classification



The Soulou spoil heap - Geotechnical classification





Basic statistics – physical parameters

Parameter	Grain size (%)			Moisture content and Atterberg Limits (%)					γ	γ_d
	Gravels	Sands	Fines	w	LL	PL	PI	LI	(kN/m^3)	(kN/m^3)
Population	128	128	128	132	104	104	104	104	129	129
Mean	11	33	57	39	51	34	16	0.3	16.9	12.6
Median	3	31	61	37	53	56	15	0.3	16.5	11.9
St. Dev.	14	12	17	18	12	12	7	0.7	2.2	3.1
COV (%)	133	38	30	45	24	34	44	253	13	24
min	0	6	9	9	22	16	3	-1.8	12.6	7.0
max	60	91	94	81	84	62	37	1.9	22.3	20.3
range	60	84	[illegible]	72	62	46	34	3.7	9.7	13.2

Plastic limit	9-29	Lee et al. (1983)
Plastic limit (clays)	3-20	Lacasse and Nadim (1996)
Plastic limit (clays, silts)	6-30	Baecher and Christian (2003)
Plasticity index	7-79	Lee et al. (1983)

Basic statistics – strength parameters

Parameter	φ' (°)	c' (kPa)
Population	43	43
Mean	25.2	17.6
Median	26.7	11.7
St. Dev.	6.8	18.7
COV (%)	27	106
min	8.2	0
max	39.2	72.4
…ge	31.0	72.4

Angle of friction (various soils)	9	Lumb (1966)
Angle of friction (sands)	5-15	Lee et al. (1983)
Angle of friction (sands)	2-5	Lacasse and Nadim (1996)
Angle of friction (sands)	5-15	Lumb (1974)
Angle of friction (clays)	12-56	Lee et al. (1983)
Angle of friction (clays)	40	Kotzias et al. (1993)
Angle of friction (alluvial soils)	16	Wolff (1996)

Basic statistics – strength parameters

Parameter	φ' (°)	c' (kPa)
Population	43	43
Mean	25.2	17.6
Median	26.7	11.7
St. Dev.	6.8	18.7
COV (%)	27	106
min	8.2	0
max	39.2	72.4
range	31.0	72.4

Cohesion (undrained, clays)	20-50	Lee et al. (1983), Lumb (1974)
Cohesion (undrained, sands)	25-30	Lee et al. (1983)

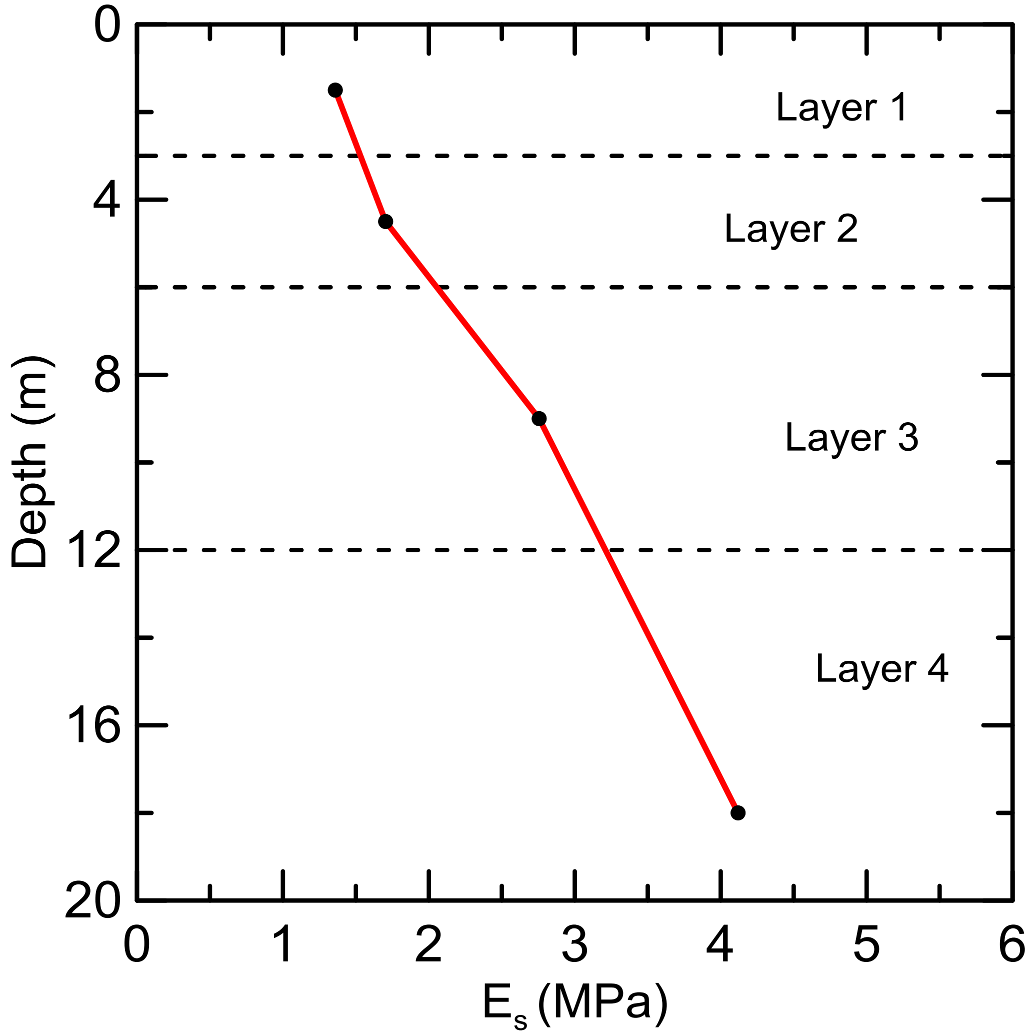
Basic statistics – compressibility parameters

Parameter	C_c (-)	C_r (-)	E_s (MPa) $\Delta\sigma_v$ = 50kPa	E_s (MPa) $\Delta\sigma_v$ = 100kPa	E_s (MPa) $\Delta\sigma_v$ = 200kPa	E_s (MPa) $\Delta\sigma_v$ = 400kPa
Population	61	61	61	61	61	61
Mean	0.216	0.037	1.9	2.7	4.0	6.4
Median	0.194	0.037	1.6	2.4	3.5	6.2
St. Dev.	0.176	0.030	1.35	1.4	1.8	2.7
COV (%)	38	41	58	51	45	42
min	0.064	0.013	0.6	1.0	1.7	3.2
max	0.398	0.090	5.7	8.1	11.3	18.7
range	0.334	0.077	5.1	7.1	9.6	15.6

Compressibility	18-73	Lee et al. (1983)
Compressibility (all soils)	25-30	Lumb (1974)
Compression, recompression index (c_c, c_r)	25-50	Lumb (1974)

Increasing Oedometric Modulus (E) with Depth

$$E_s(MPa) = 0.18 \cdot z + 1$$



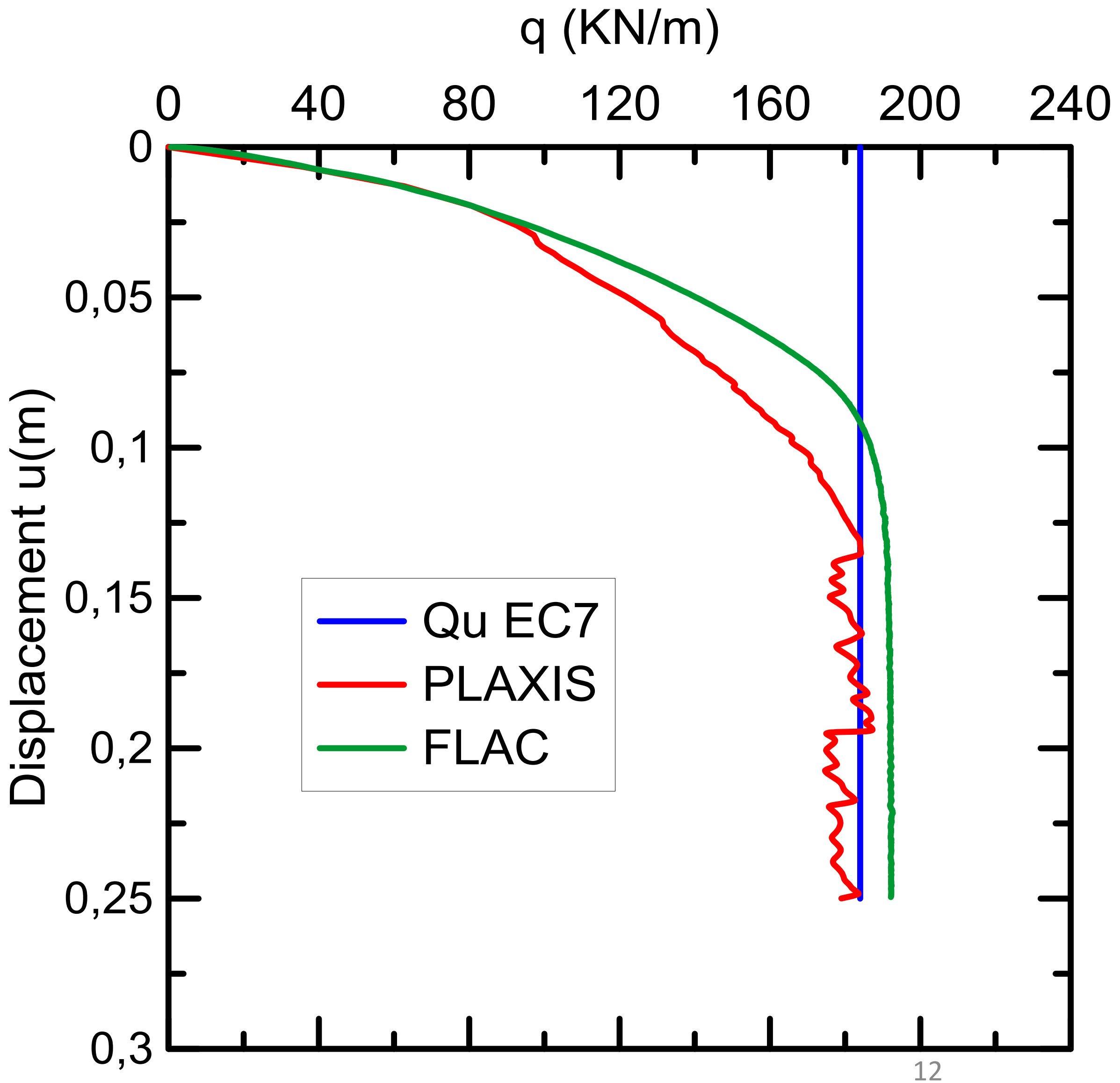
Bearing Capacity / Load - Displacement Curve

Bearing Capacity :

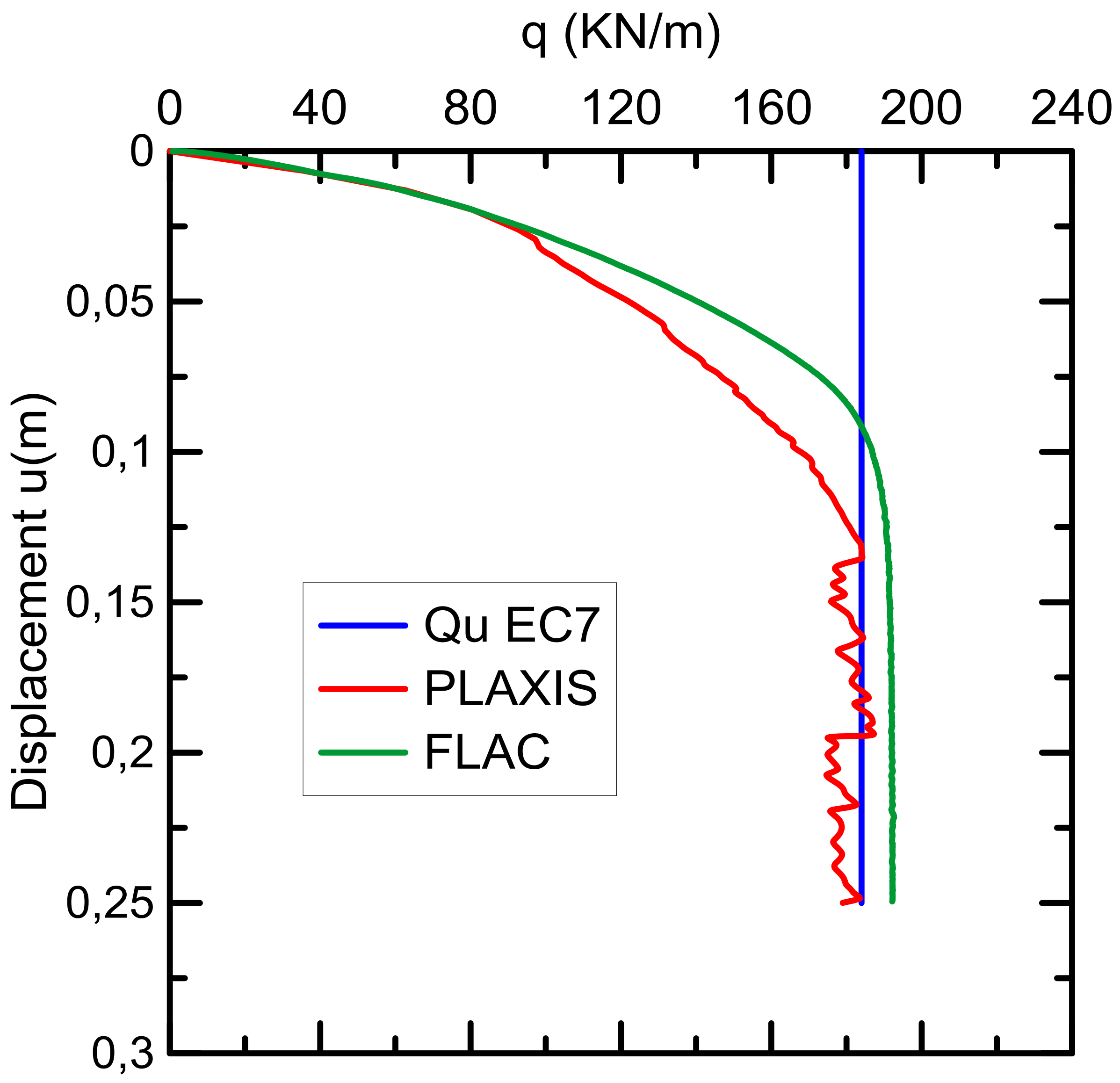
- EC7 : Q=184 KN
- FLAC: Q=190 KN
- PLAXIS: Q=185 KN

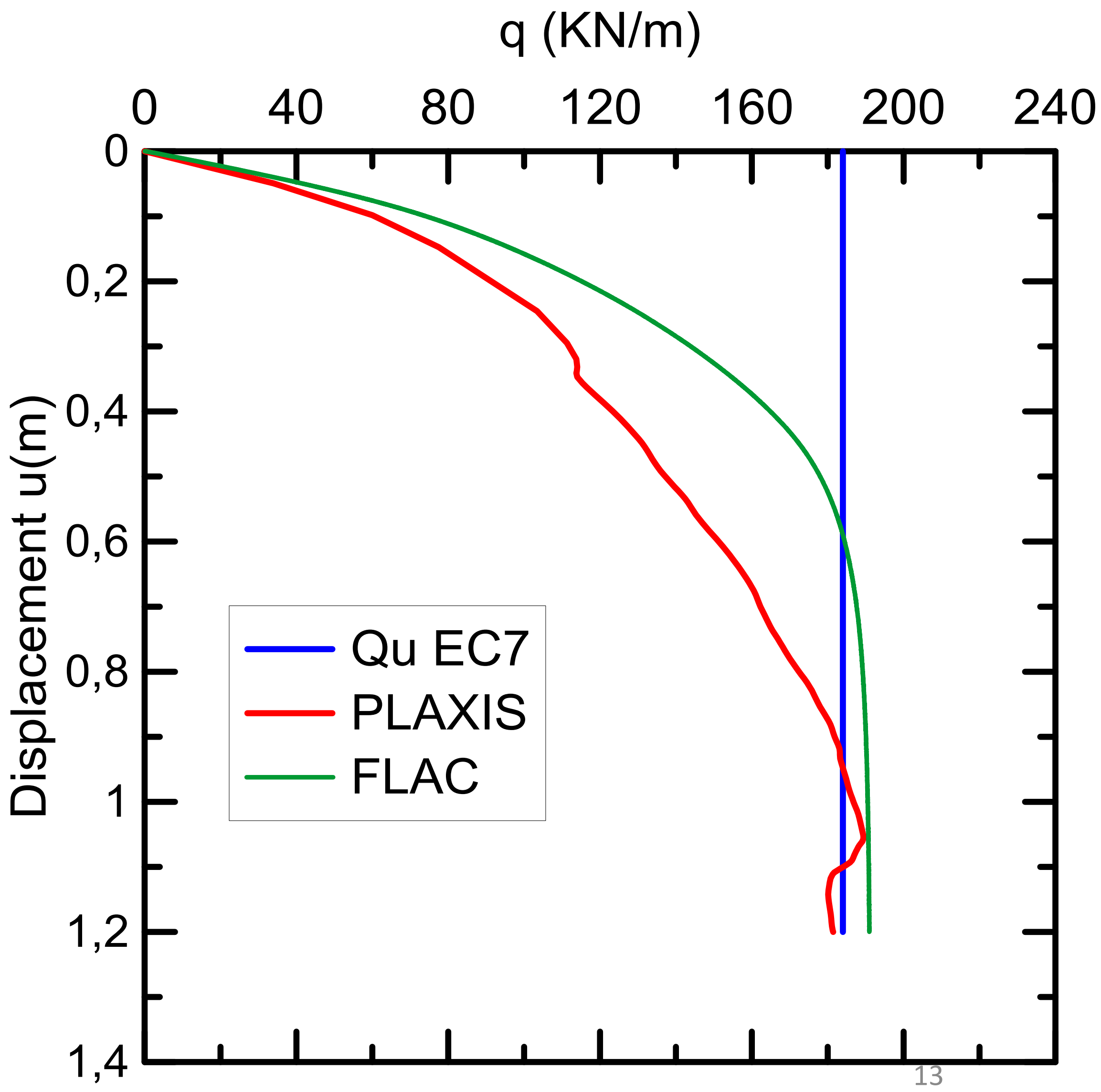
<u>Comments:</u>

- Regarding the bearing capacity, numerical results match almost perfectly with the analytical solution
- The maximum settlements are approximately the same (0,12m) for both software.

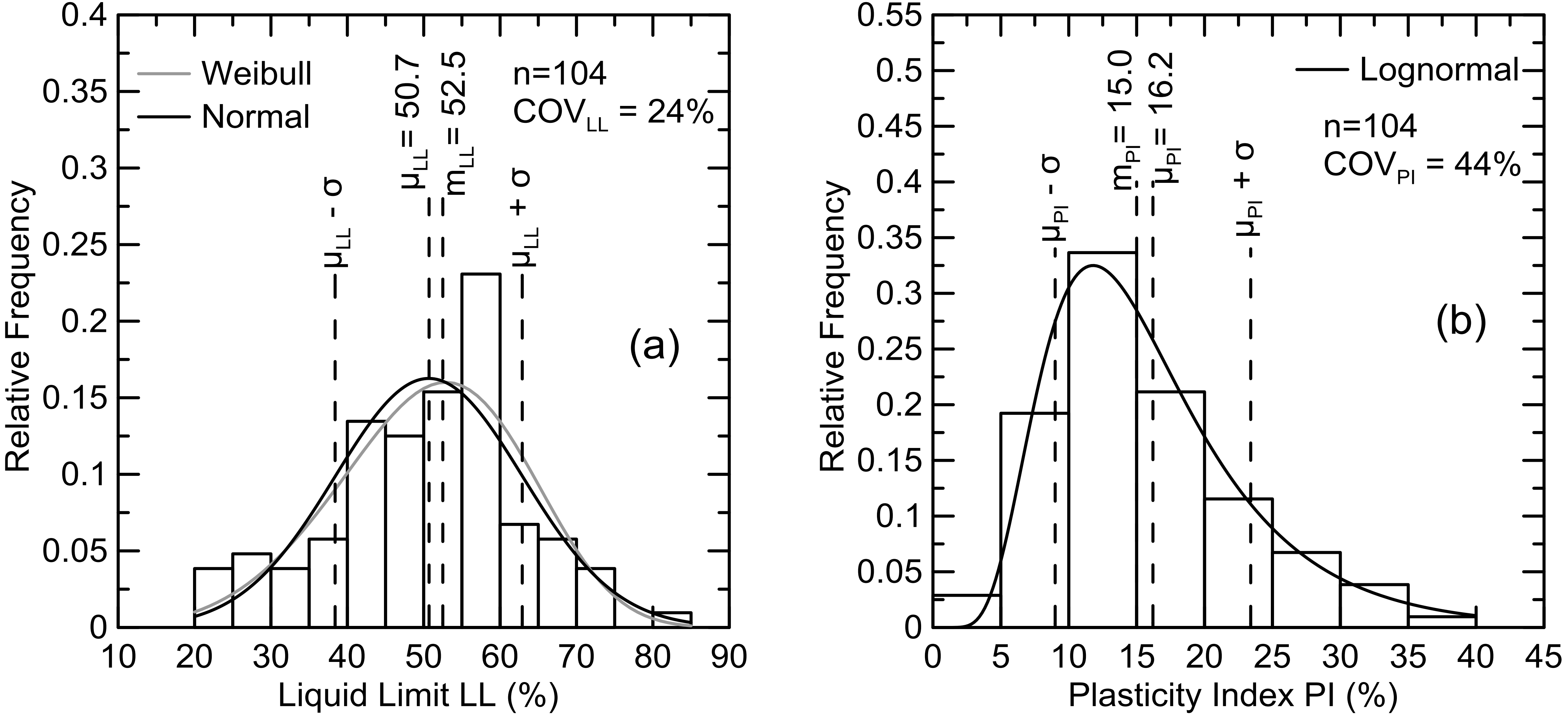


Increasing Oedometric Modulus (E) with Depth

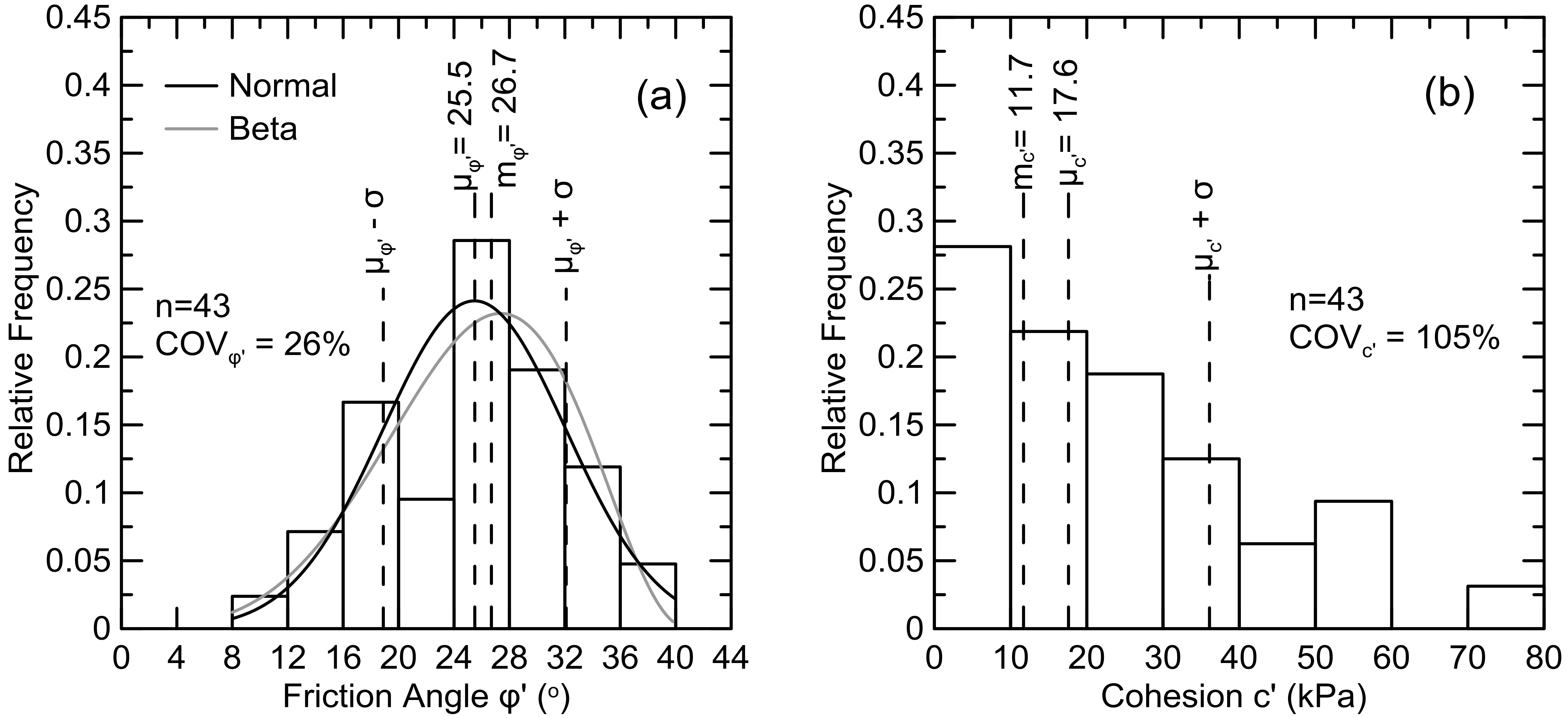




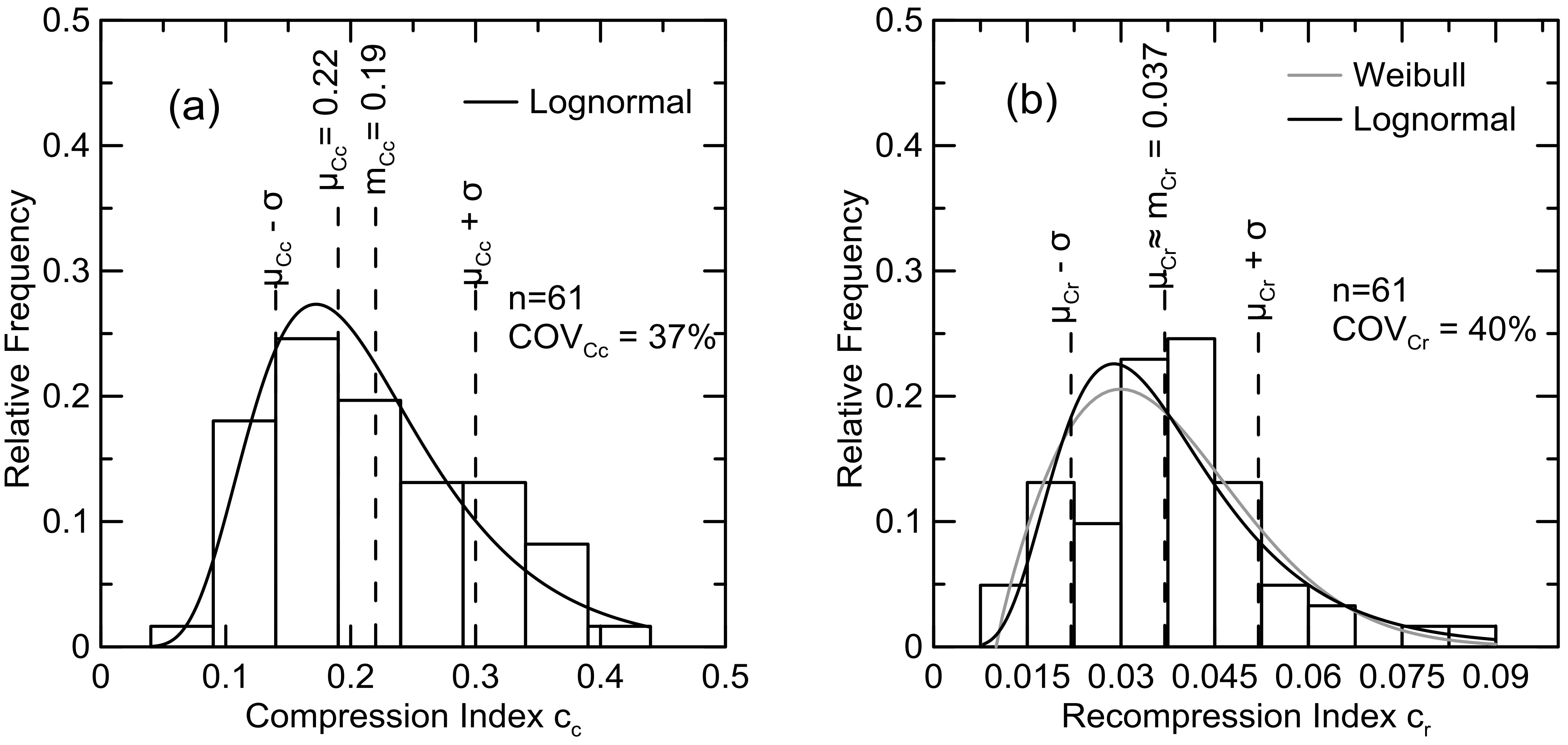
Statistical distribution – physical parameters



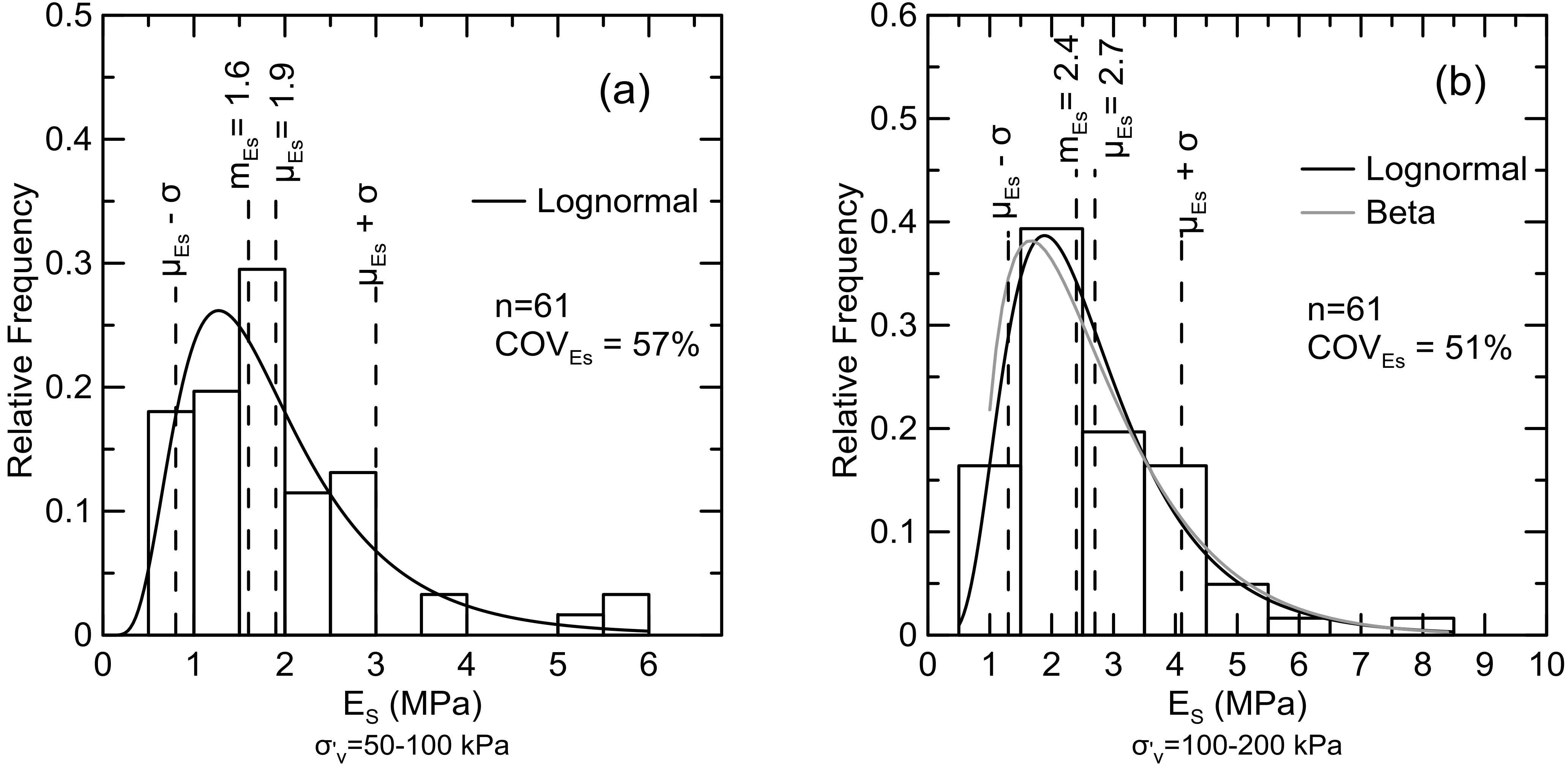
Statistical distribution – strength parameters



Statistical distribution – compressibility parameters



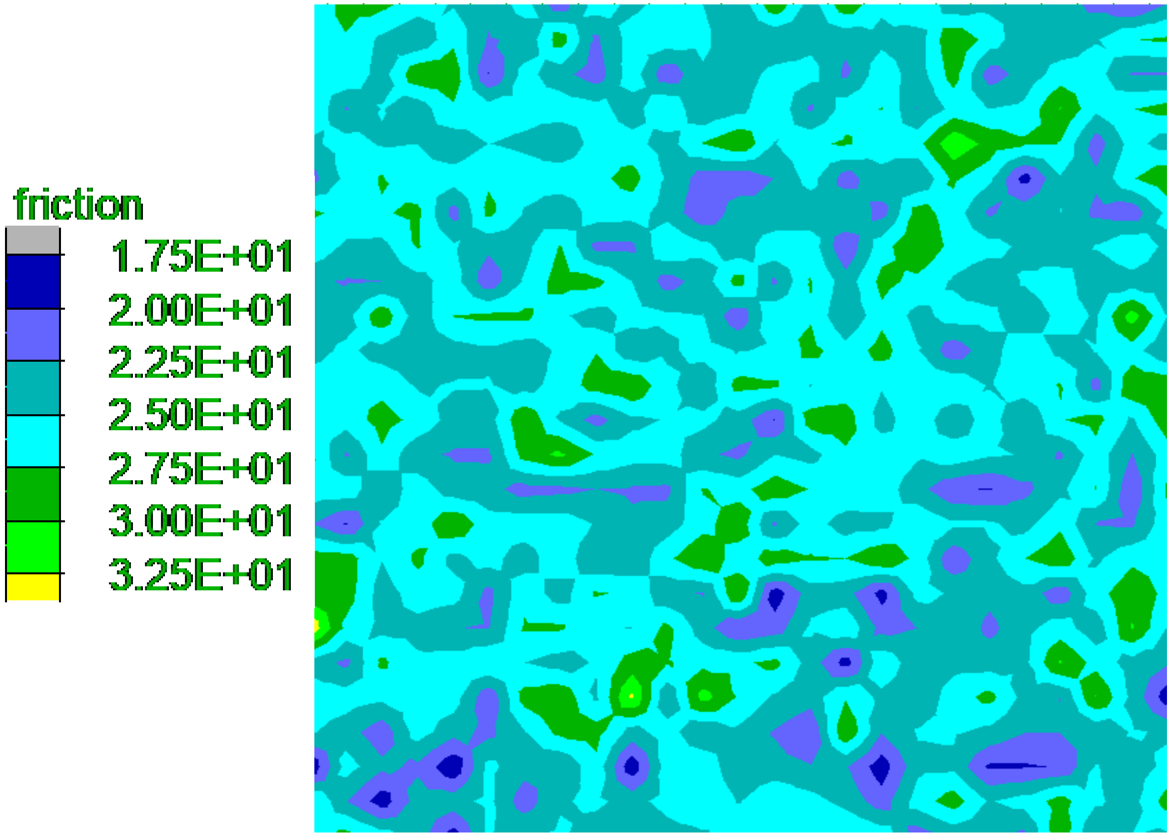
Statistical distribution – compressibility parameters



Spatial variability

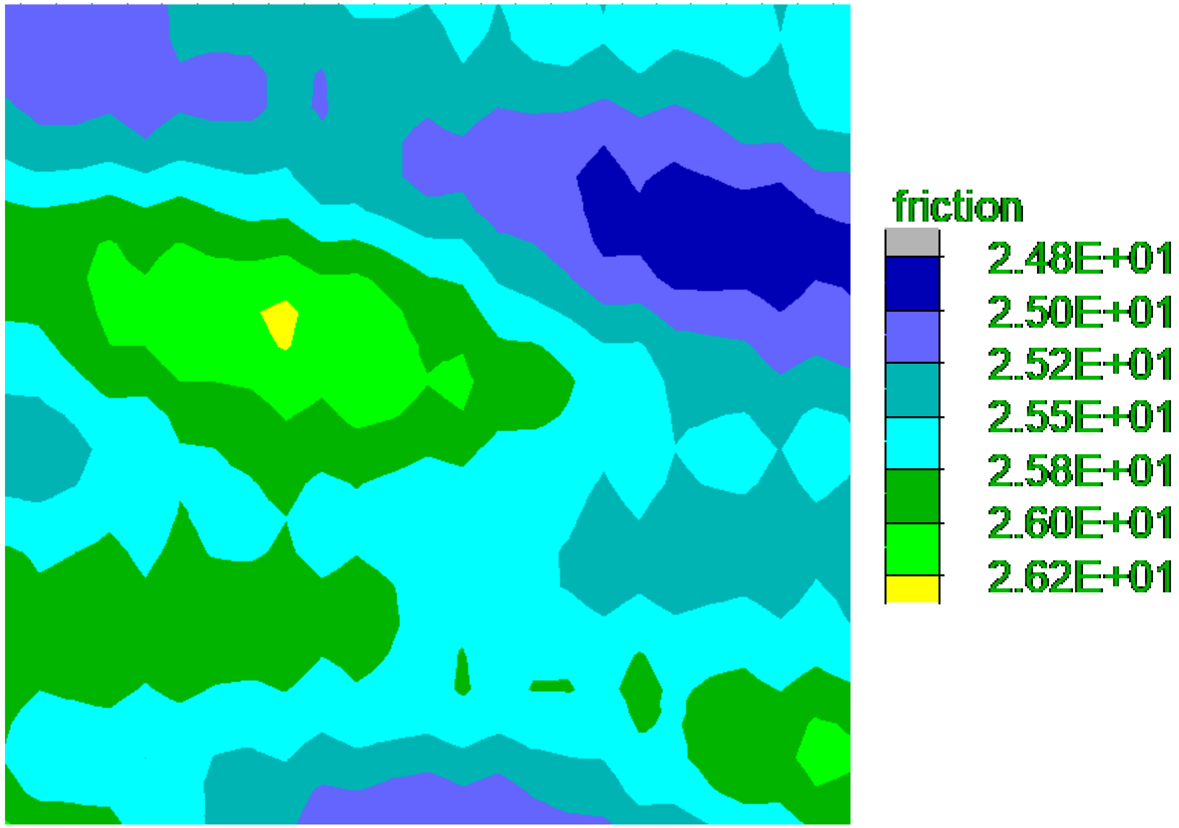
The effect of the correlation length is crucial to the definition of the spatial random field. The following figures show two extreme cases illustrating the effect. All other properties are the same.

Small Correlation Length l=0.001



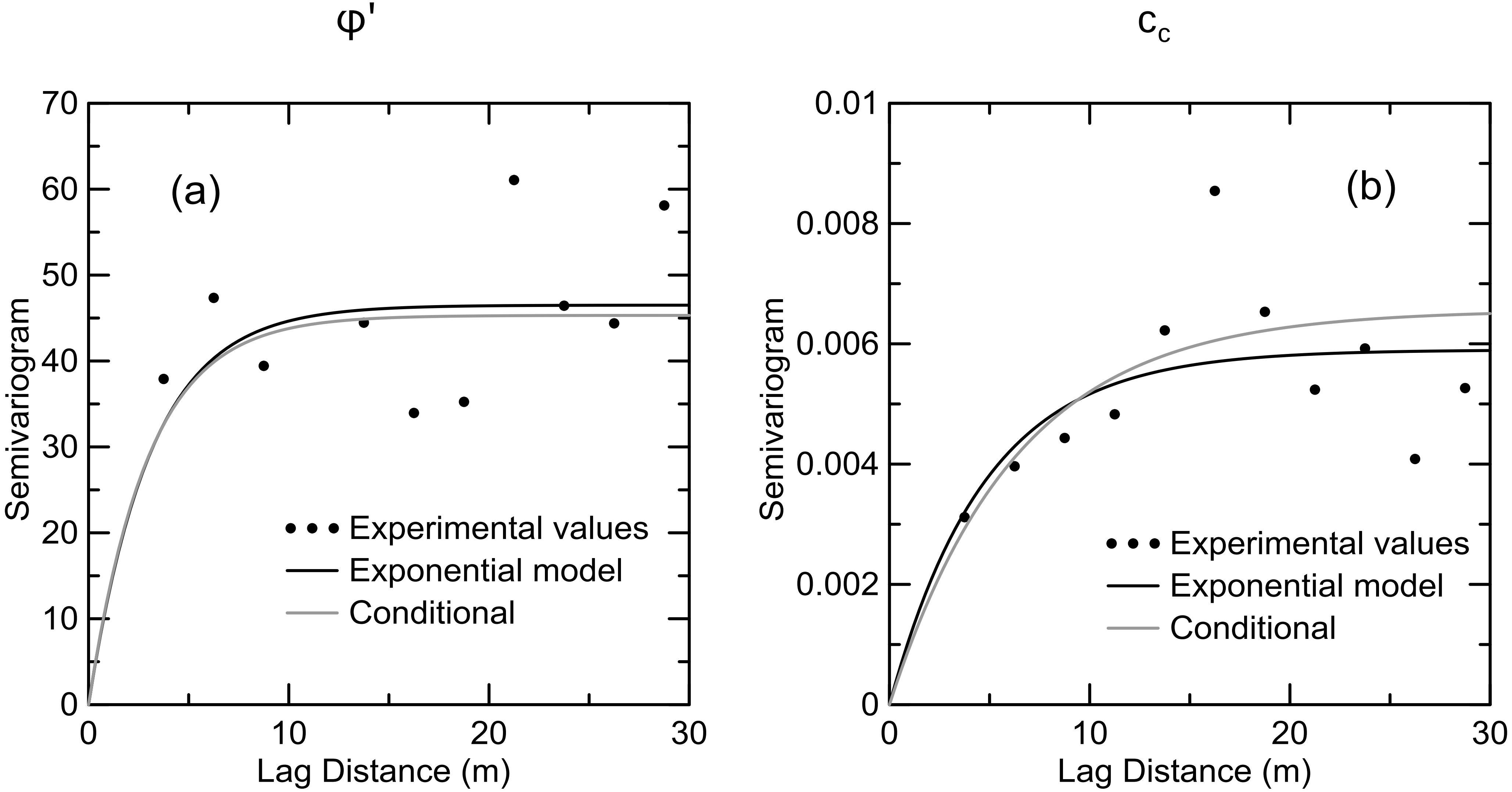
Great Spatial Variations

Big Correlation Length l=1000

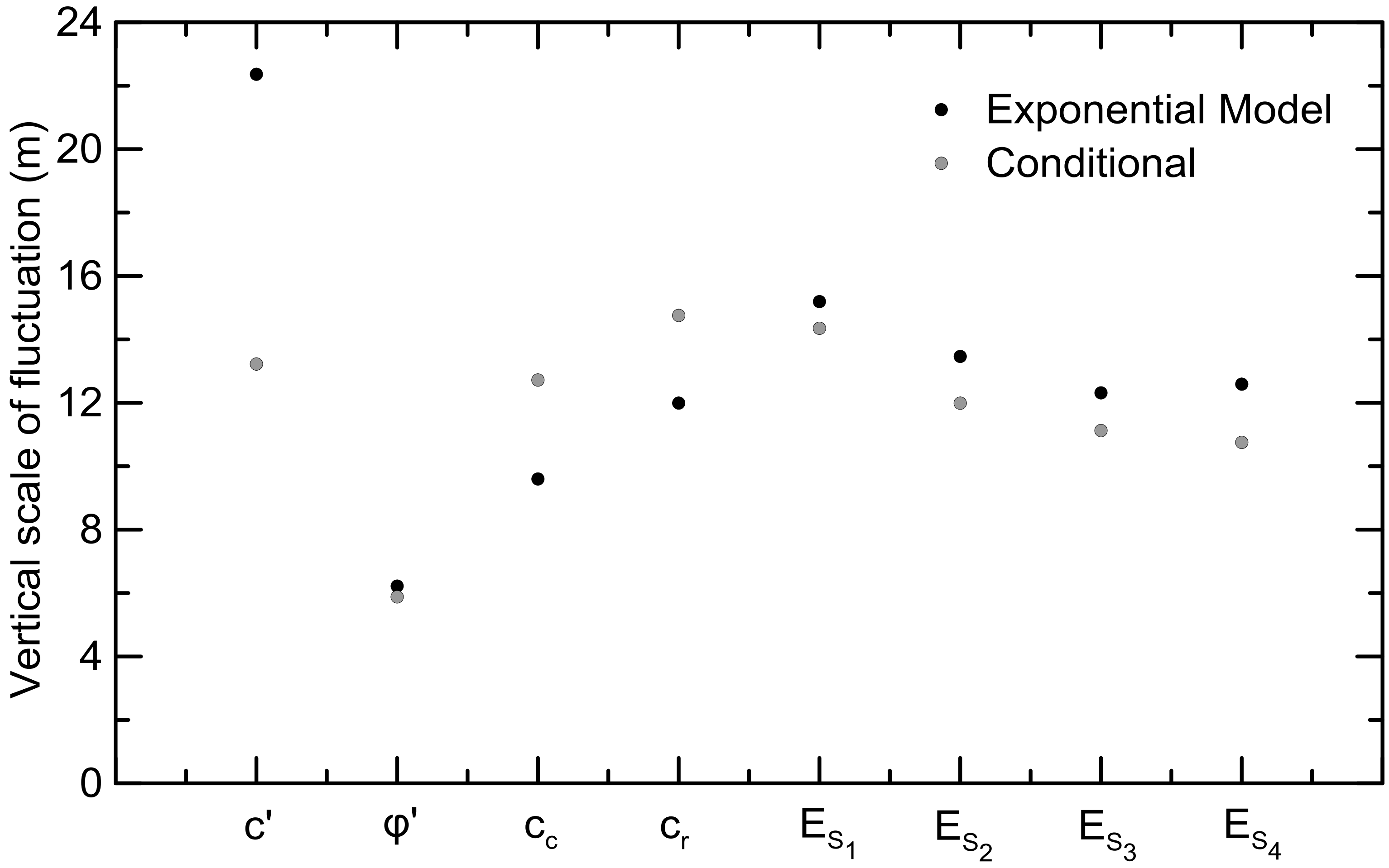


Small Spatial Variations

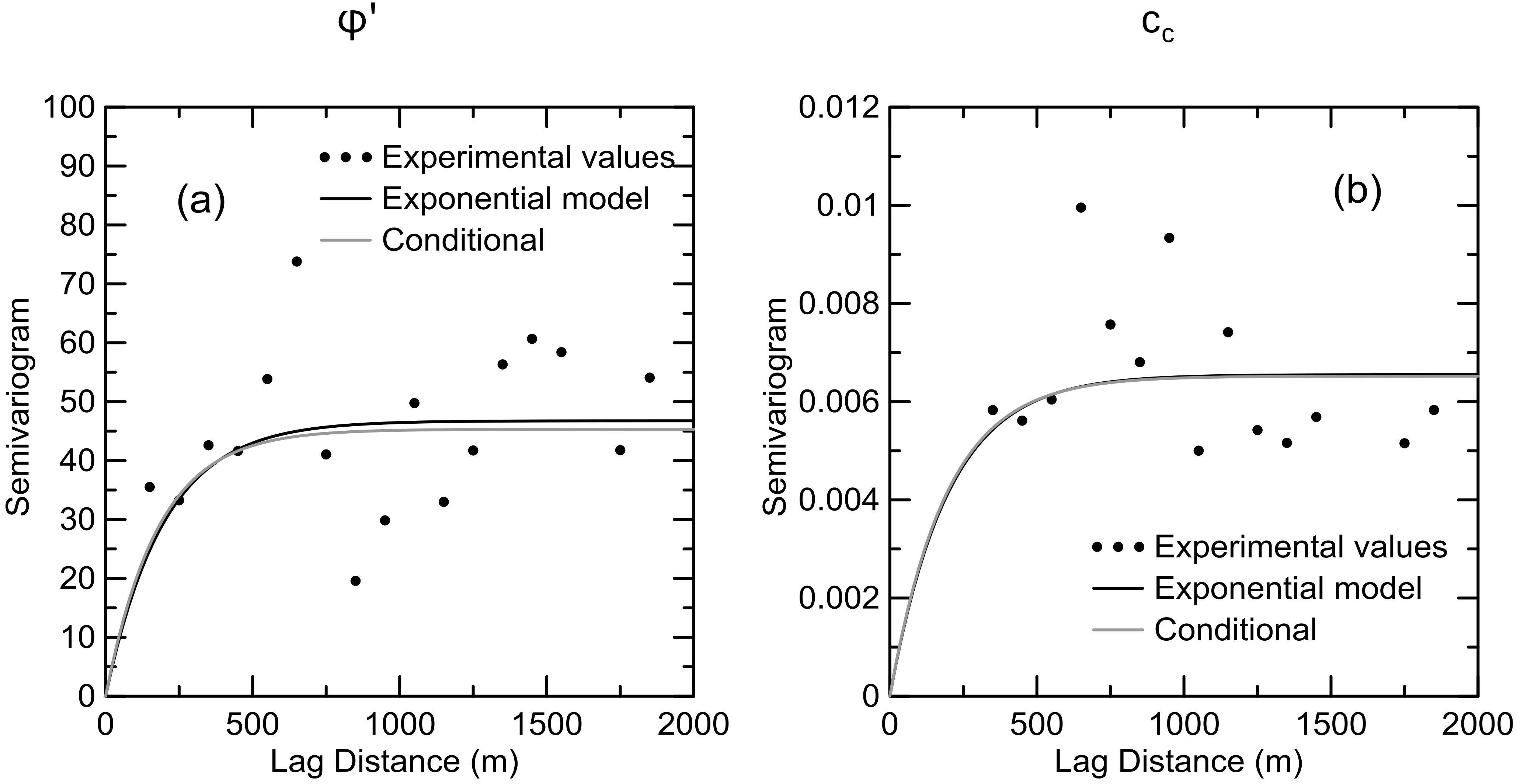
Vertical correlation length



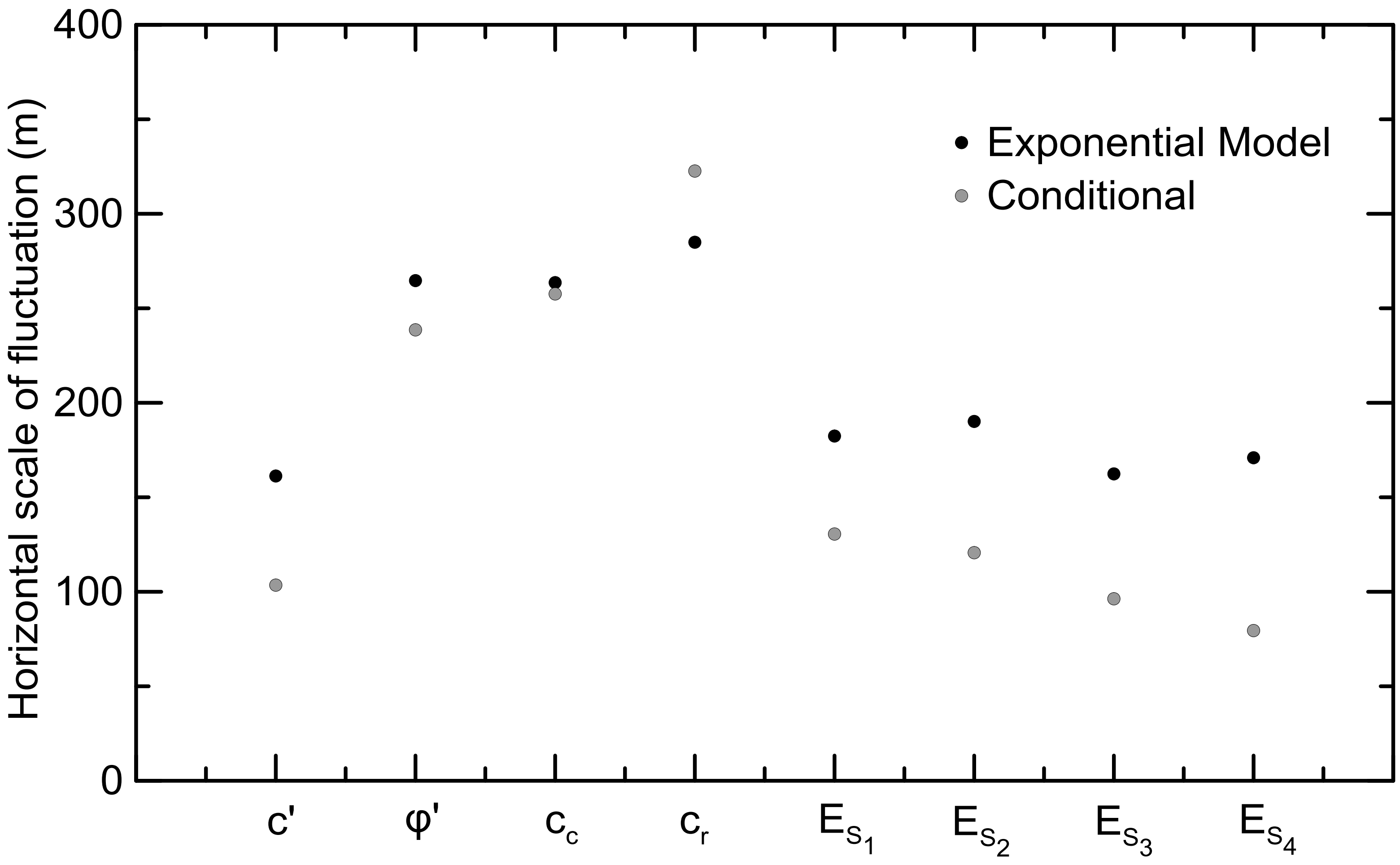
Vertical correlation length



Horizontal correlation length



Horizontal correlation length



Conclusions

- Effective friction angle: symmetric, wide-ranged distribution, many observations at its tails, Weibull distribution.
- Effective cohesion: unusual distribution with a significant number of low or zero values and a large tail up to 80 kPa; none of the tested probability distributions
- Compression index: right-skewed distribution with a significant range; lognormal
- Recompression index presents a symmetric and narrow distribution; Weibull.
- Constraint modulus: narrow distribution with most observations within one standard deviation from the mean; lognormal (50-100 kPa), Beta (100-200 and 200-400 kPa), and Weibull (400-800 kPa).

Conclusions

- Vertical direction: strongly correlated elements for distances larger than 10m, reaching even 20m
- A single representative value and a homogeneous soil would probably be unrealistic
- Horizontal plane: scales of fluctuation reaching up to 285m.
- Horizontally uniformly distributed for most engineering applications one characteristic value of the property

Thank you !

WP3 Task 3.3: Numerical simulation of alternative design cases & analysis of design methodologies

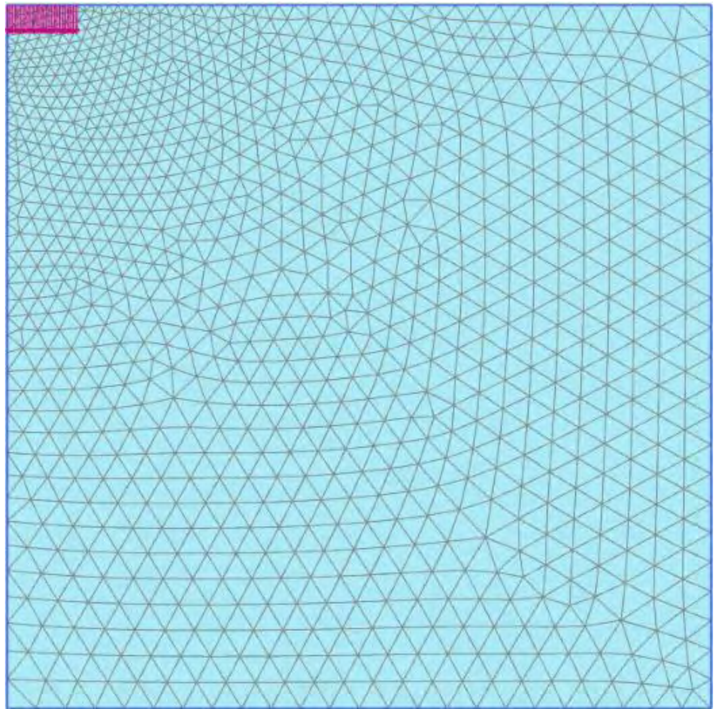
FLAC 2D vs PLAXIS 2D – Validation

Problem: Strip Footing on the surface of Soulou spoil heap.

Goal : Determine the bearing capacity of the footing using the EC7, FLAC 2D and PLAXIS 2D. Compare the different results and the Load-Displacement curves derived by the software.

- Symmetric Problem → Half problem will be simulated.
- Uniform Spoil Profile

Mesh Plaxis



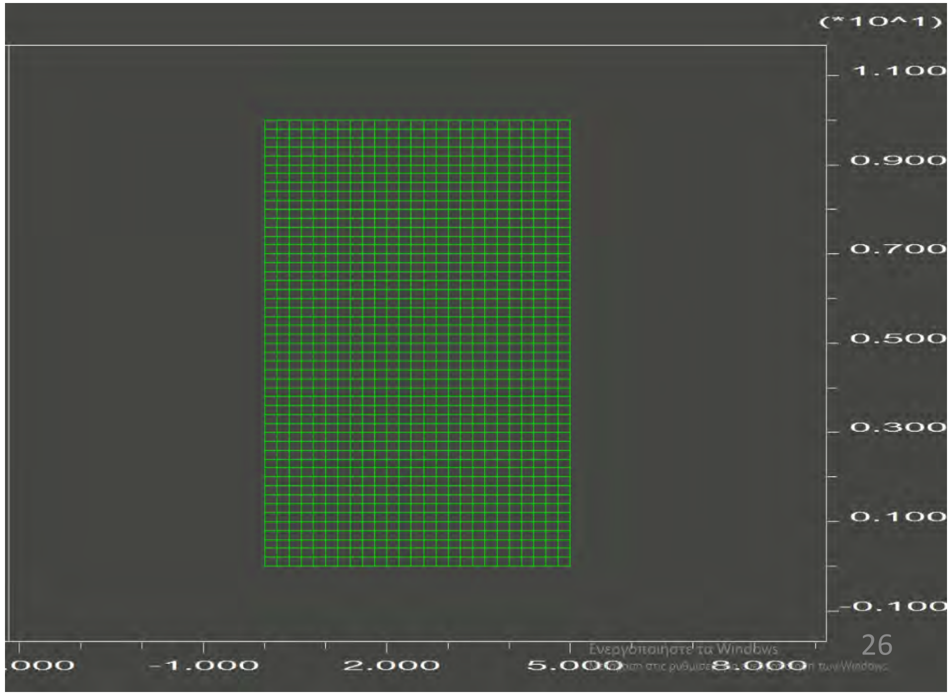
Mohr-Coulomb Model

- γ=16,9 KN/m3
- ϕ=25,2 o
- c=5 kPa
- E=10 Mpa

Strip Footing

B=1m

Mesh Flac

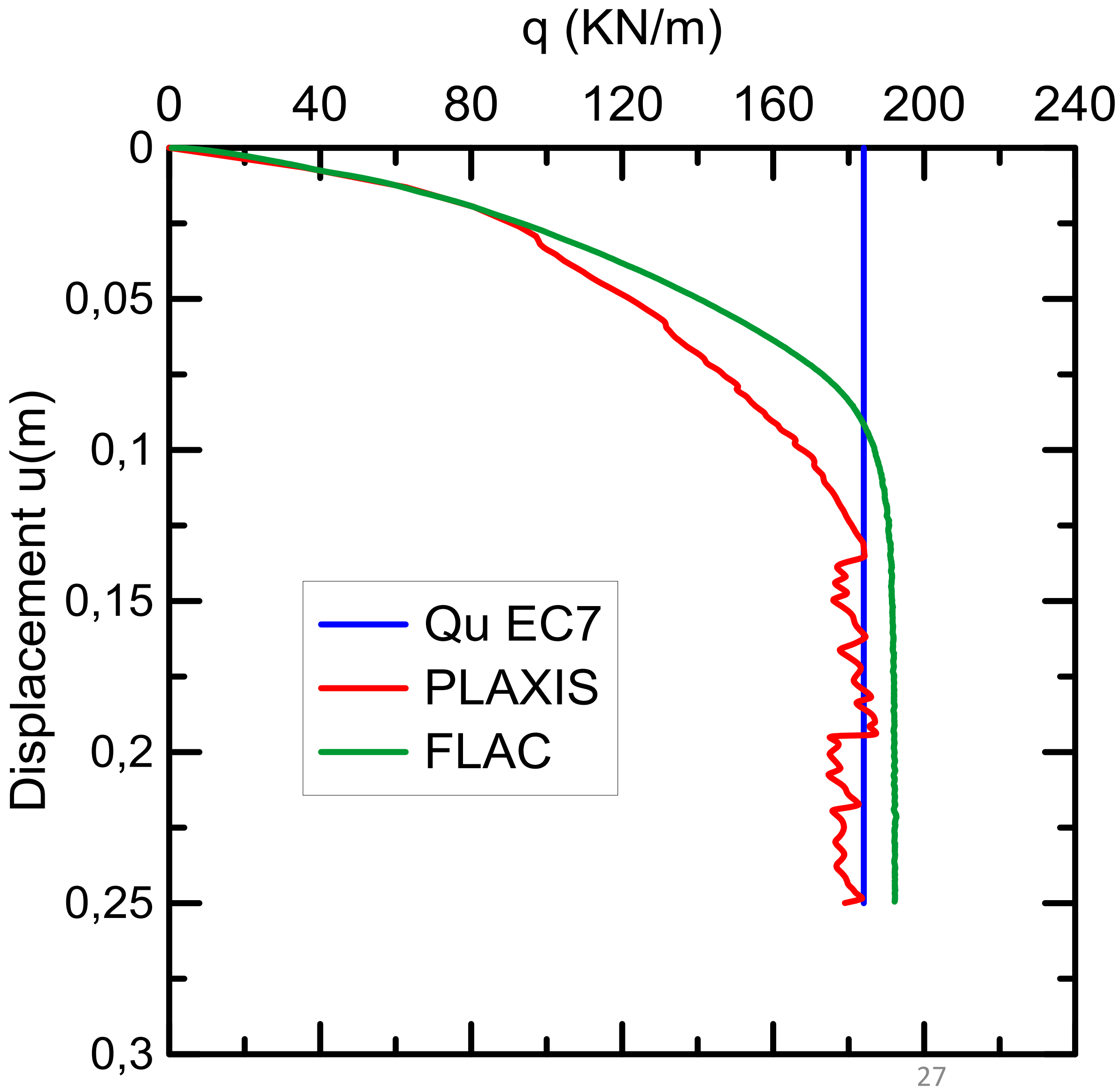


WP3 Task 3.3: Numerical simulation of alternative design cases & analysis of design methodologies

Bearing Capacity / Load - Displacement Curve

Bearing Capacity :

- EC7 : Q=cNc+qNq+(1/2)γBNγ =184 KN
- FLAC: Q=190 KN
- PLAXIS: Q=185 KN



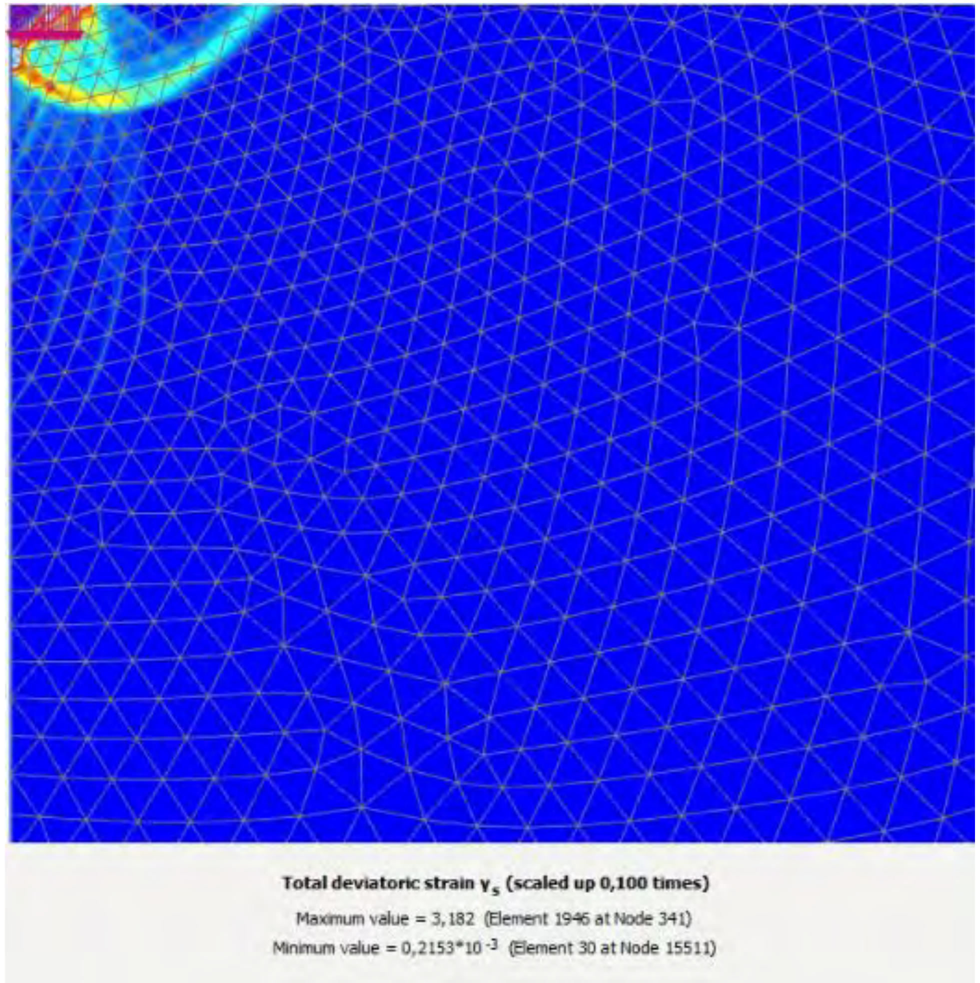
Comments:

- Regarding the bearing capacity, numerical results match almost perfectly with the analytical solution
- The maximum settlements are approximately the same (0,12m) for both software.
- The inclination difference between the two curves is justified by the different "evolution" of plastic points (see next slides)

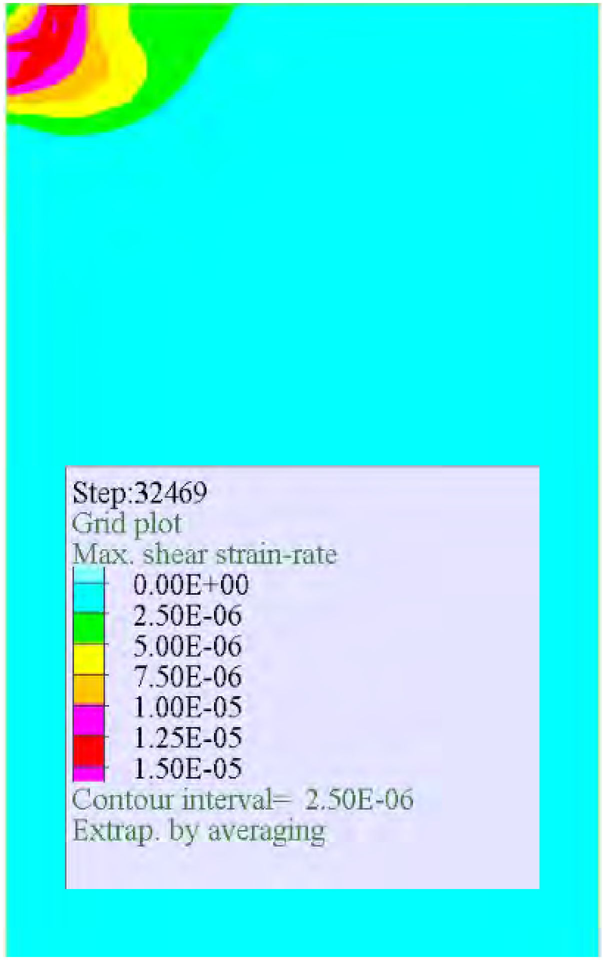
WP3 Task 3.3: Numerical simulation of alternative design cases & analysis of design methodologies

FLAC 2D vs PLAXIS 2D – Validation

Shear Strains



PLAXIS

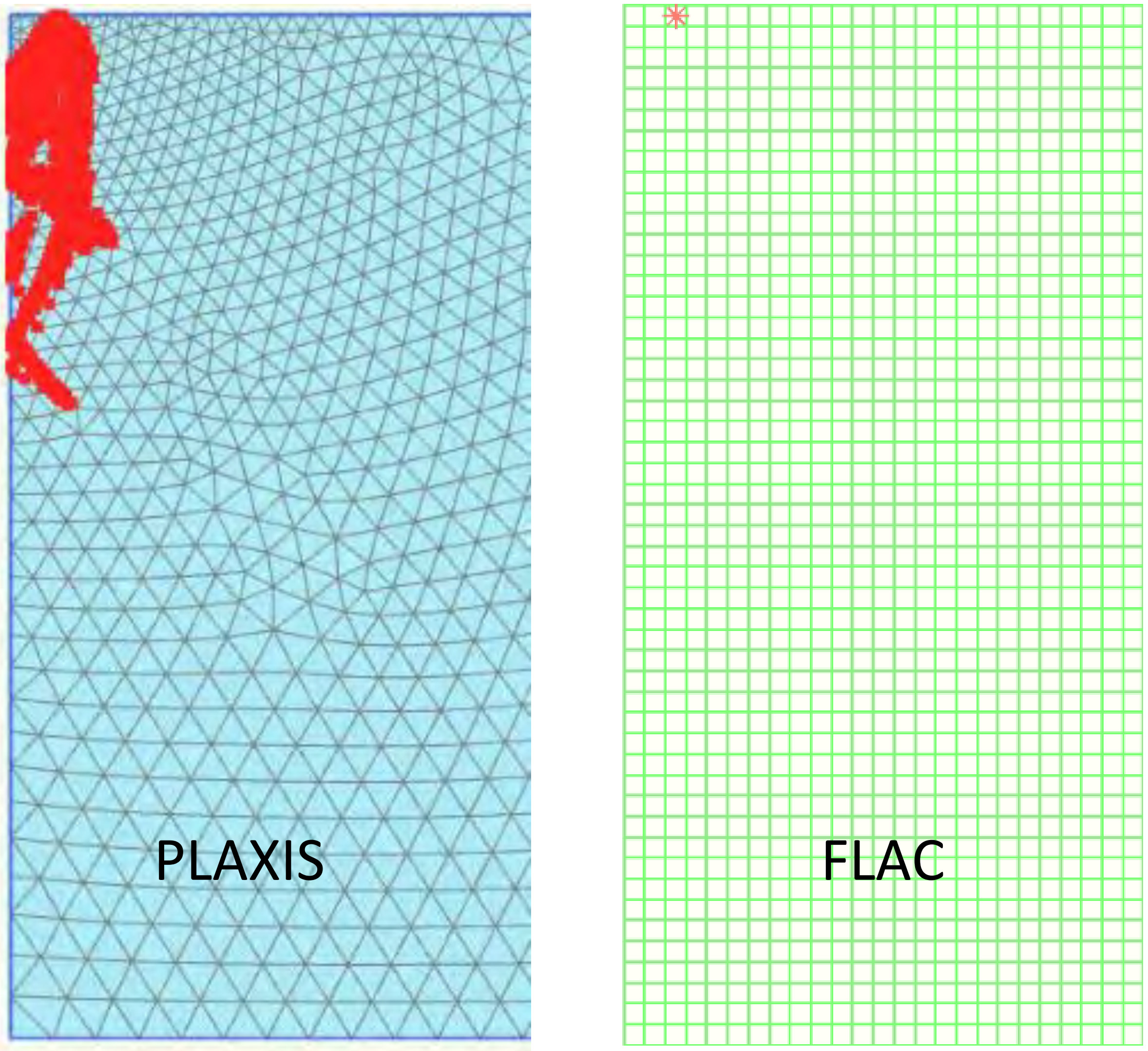


FLAC

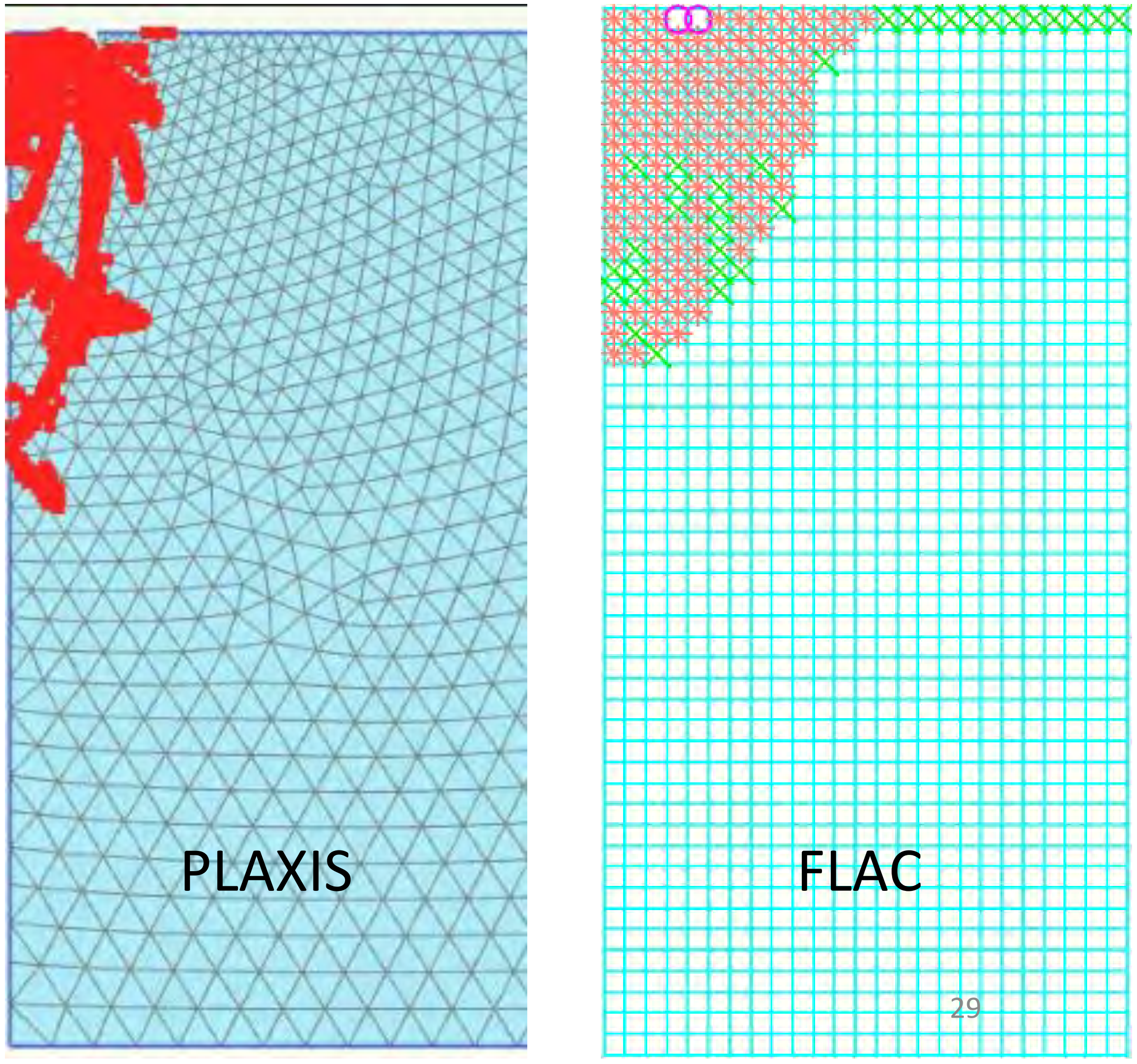
WP3 Task 3.3: Numerical simulation of alternative design cases & analysis of design methodologies

FLAC 2D vs PLAXIS 2D – Validation

Plastic Points at 5cm displacment



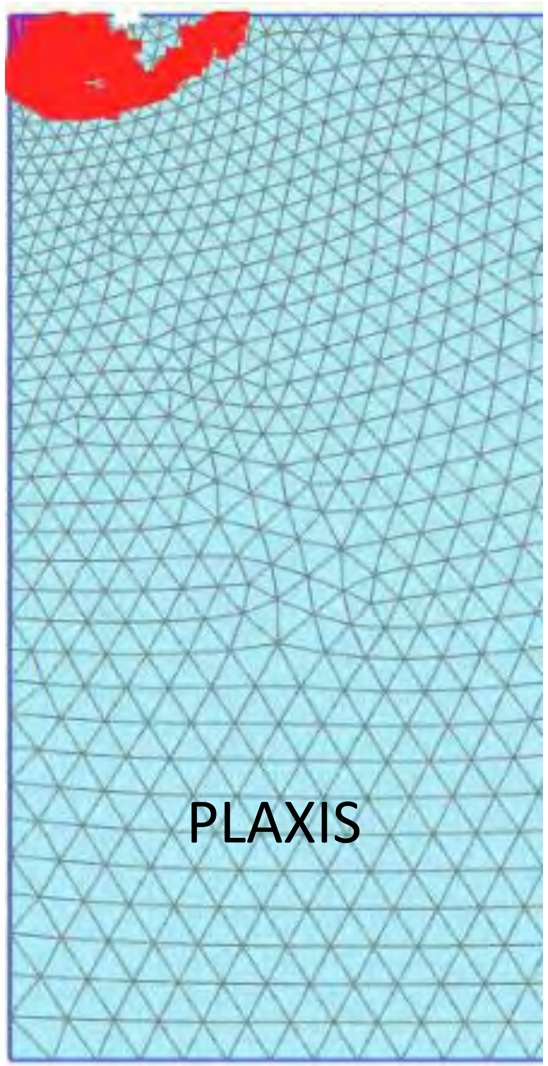
Plastic Points at 10cm displacment

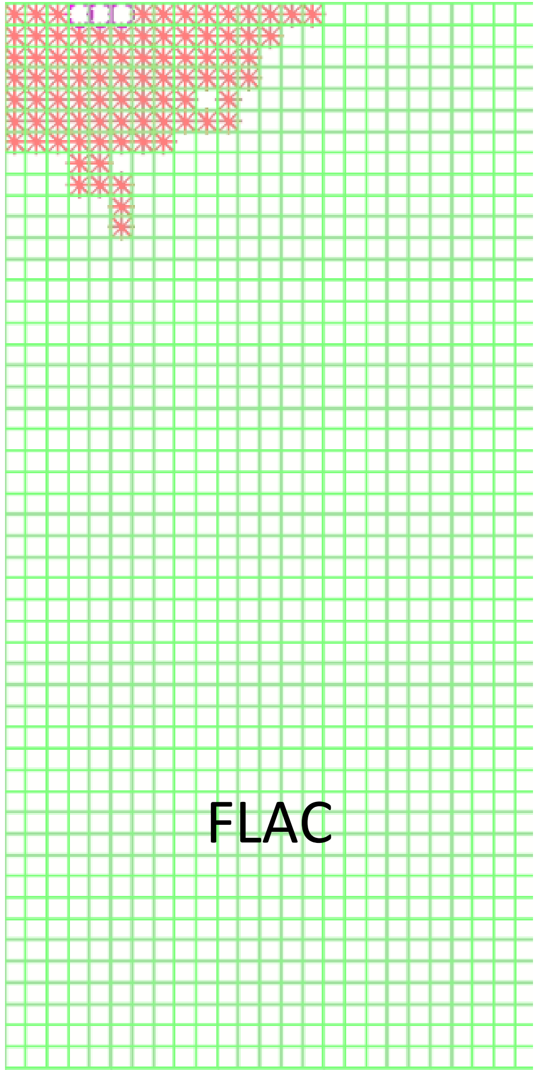


WP3 Task 3.3: Numerical simulation of alternative design cases & analysis of design methodologies

FLAC 2D vs PLAXIS 2D – Validation

Plastic Points at 25cm settlement





WP3 Task 3.3: Numerical simulation of alternative design cases & analysis of design methodologies

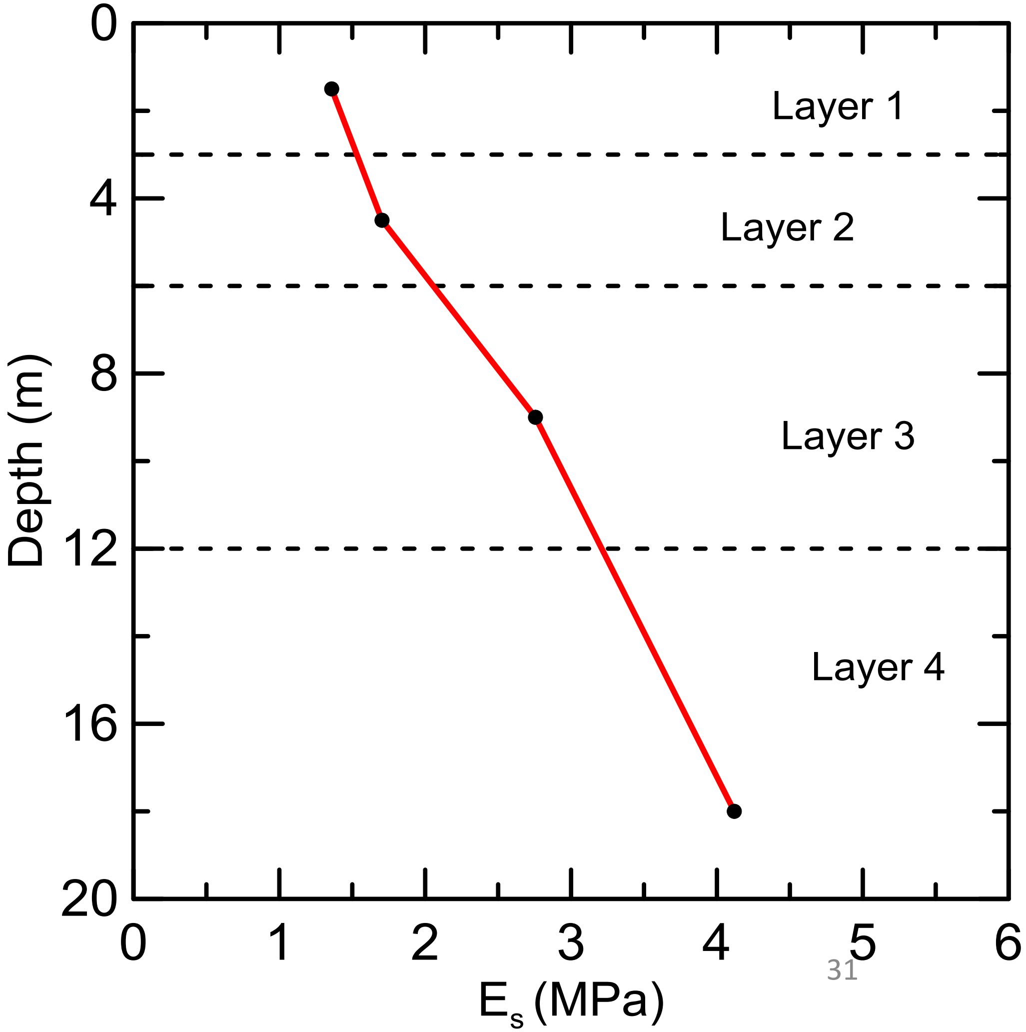
Increasing Oedometric Modulus (E) with Depth

Based on the laboratory oedometer tests conducted on samples from Soulou in different stress increments, an effort was made to see if a straight line representing the increase of the oedometric modulus (Es) with depth can be derived.

The different stress increments from the oedometer tests, were assumed to correspond to different soil layers with different Es represented by its mean value.

Samples that were in higher initial (in situ) stresses than the ones subjected in the oedometer test were excluded from this analysis in order to plot only the ones above the initial (in situ) σ_v the samples were exposed to.

$$E_s(MPa) = 0.18 \cdot z + 1$$



WP3 Task 3.3: Numerical simulation of alternative design cases & analysis of design methodologies

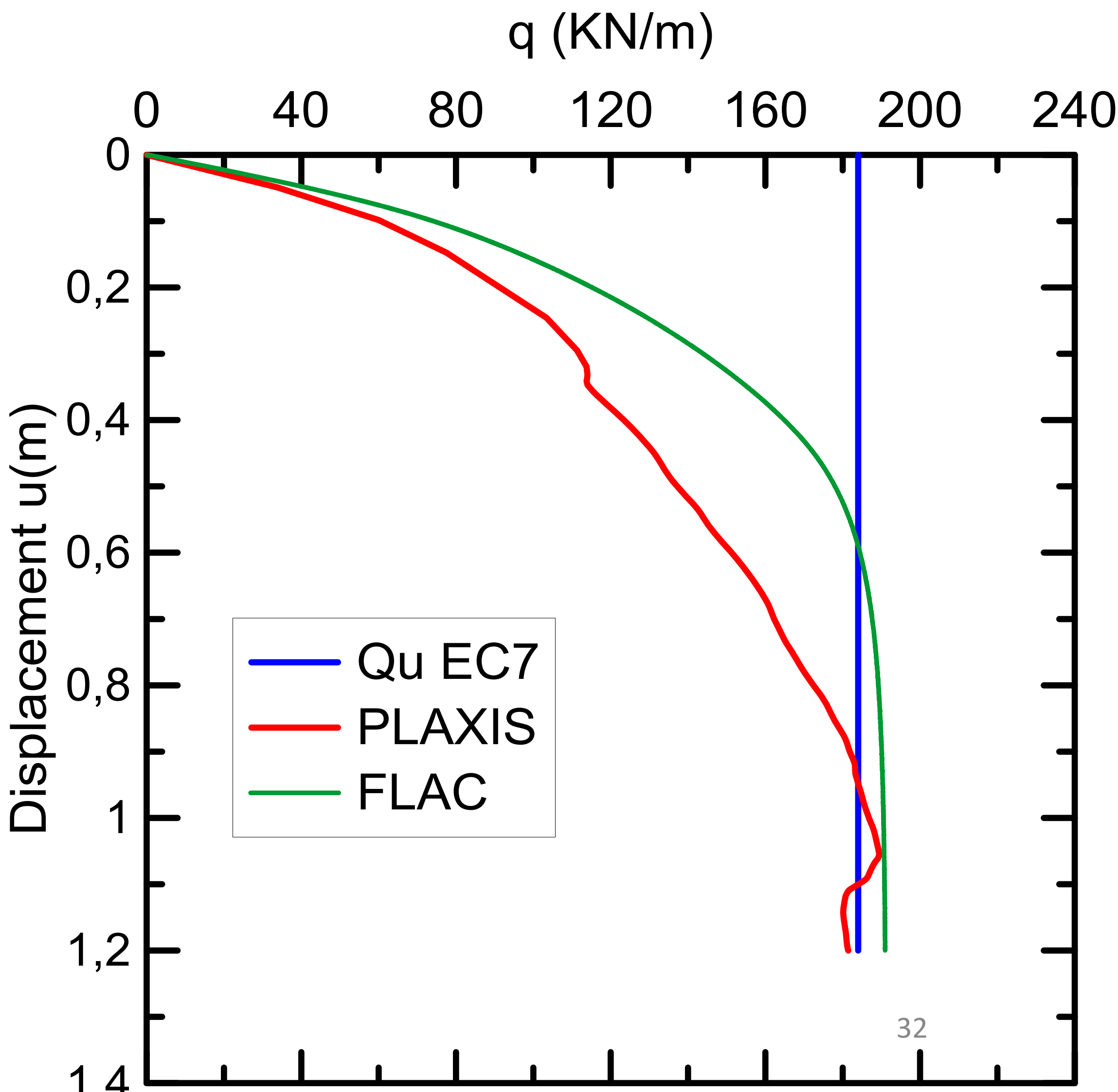
Increasing Oedometric Modulus (E) with Depth

Bearing Capacity :

- EC7 : Q=cNc+qNq+(1/2)γBNγ =184 KN
- FLAC: Q=192 KN
- PLAXIS: Q=185 KN

<u>Comments:</u>

- Regarding the bearing capacity, numerical results are very close to the analytical solution.
- FLAC gives slightly more optimistic results regarding both the maximum bearing capacithy and the maximum settlement



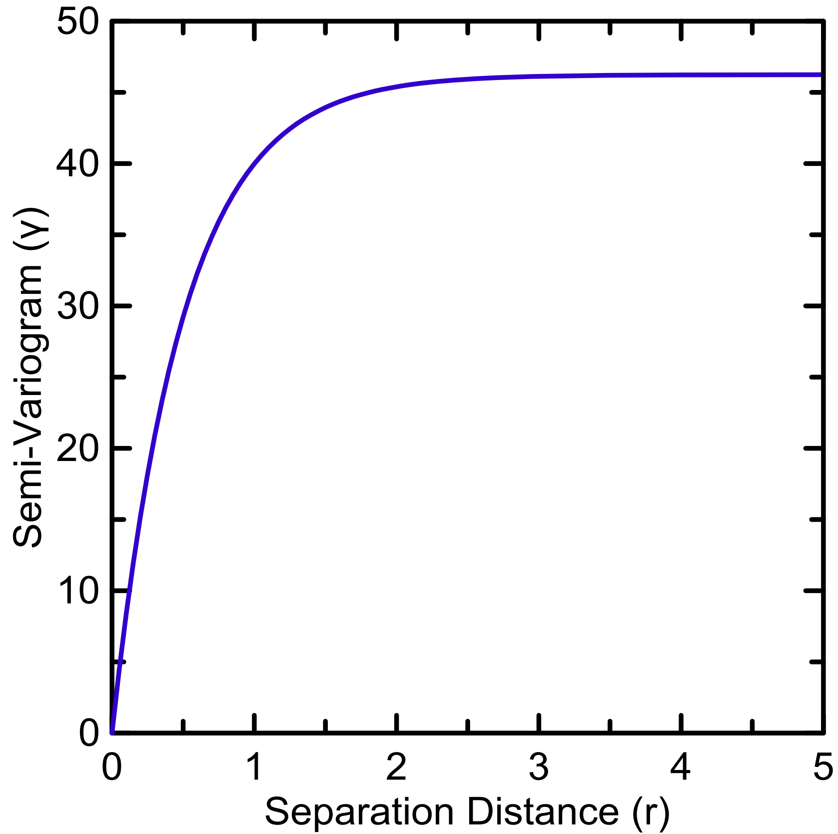
WP3 Task 3.3: Numerical simulation of alternative design cases & analysis of design methodologies

Spatial Characterization

To accurately characterize a spatially variable property the following parameters are significant:

- Probability Distribution (PDF,CDF)
- Statistical Moments (Variance, mean etc)
- Correlation length (length beyond which the properties are assumed to be uncorrelated)

The most useful tool to imprint all the needed information is the Variogram:



Variogram: Whereas the covariance is defined as the expected value of the product of two observations, the variogram (2γ) is the expected value of the squared difference and shows the decreasing dependence between two points as the distance between them grows bigger (Measure of Dis-similarity)

$$2\gamma = \mathbf{E}\left[\{\mathbf{z}(x_i) - \mathbf{z}(x_j)\}^2\right] = \boldsymbol{Var}[\mathbf{z}(x_i) - \mathbf{z}(x_j)]$$

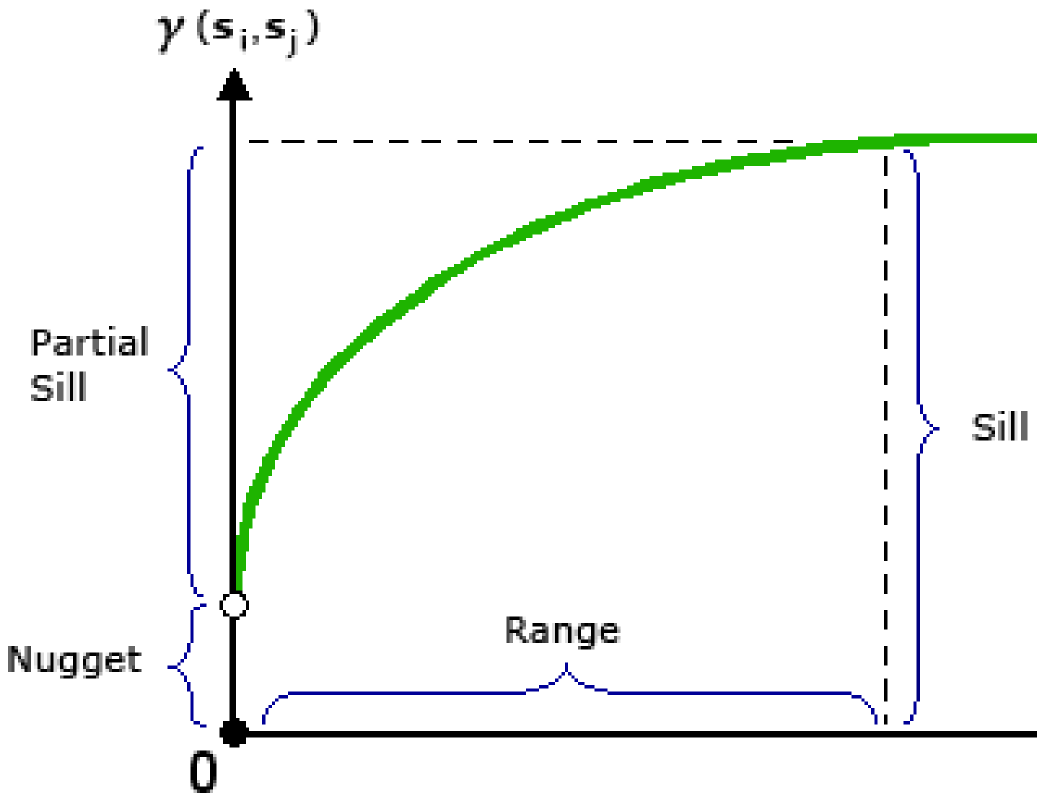
WP3 Task 3.3: Numerical simulation of alternative design cases & analysis of design methodologies

Spatial Characterization – Variogram Modelling

Variogram Equation

$$\gamma(r) = \sigma^2 \cdot \left(1 - exp\left(\frac{r}{\theta}\right)\right) + n$$

- σ^2 = variance
- r = distance between two points
- ℓ= correlation length
- cor = correlation function (Gaussian, exponential, etc.)
- n= nugget



- Sill = Limit of the variogram to infinity lag distances
- Range = The distance in which the difference of the variogram from the sill becomes negligible.
- Nugget = the height of the jump of the semivariogram at the discontinuity of the origin

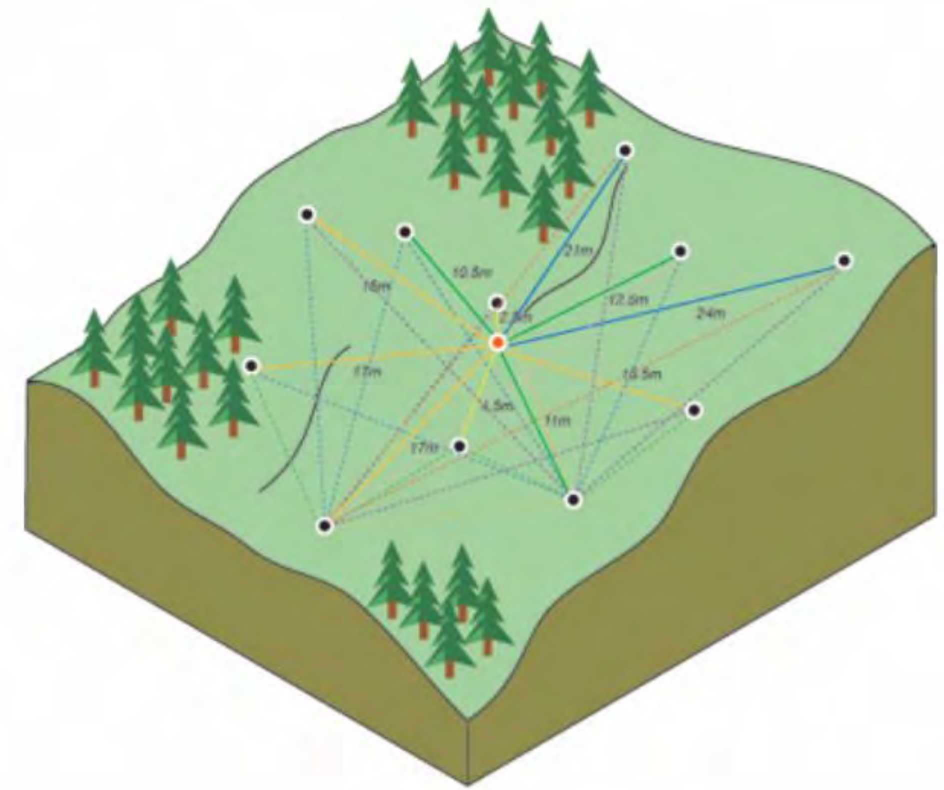
WP3 Task 3.3: Numerical simulation of alternative design cases & analysis of design methodologies

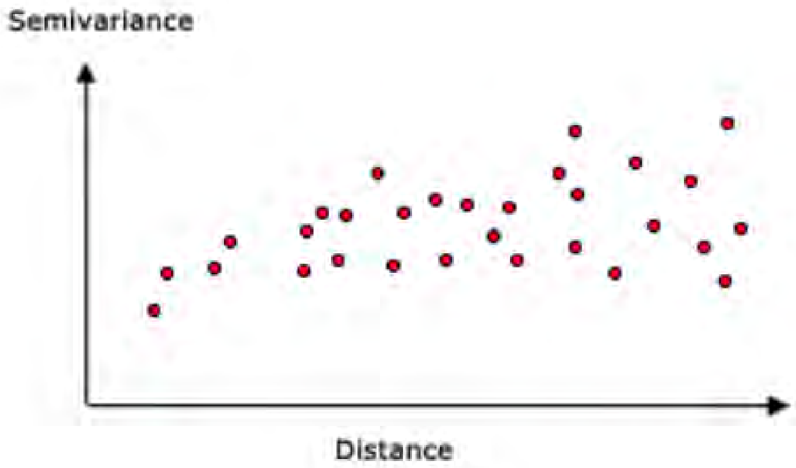
Spatial Characterization – Variogram Modelling

For the property under investigation:

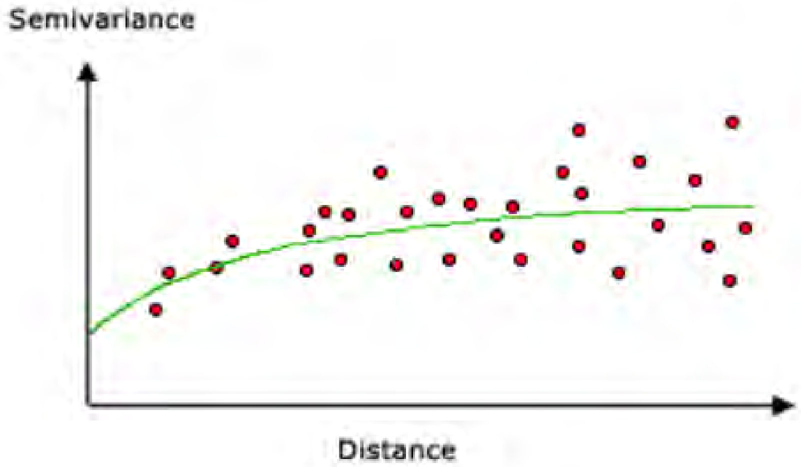
1. Calculate the squared difference between the values of each pair of locations : $\gamma(h) = 0.5 \cdot (x_i - x_j)^2$
2. Plot the points in order to create the empirical semi-variogram.
3. Fit a theoretical semi-variogram model (exponential, Gaussian etc.) to the empirical semi-variogram

1





2

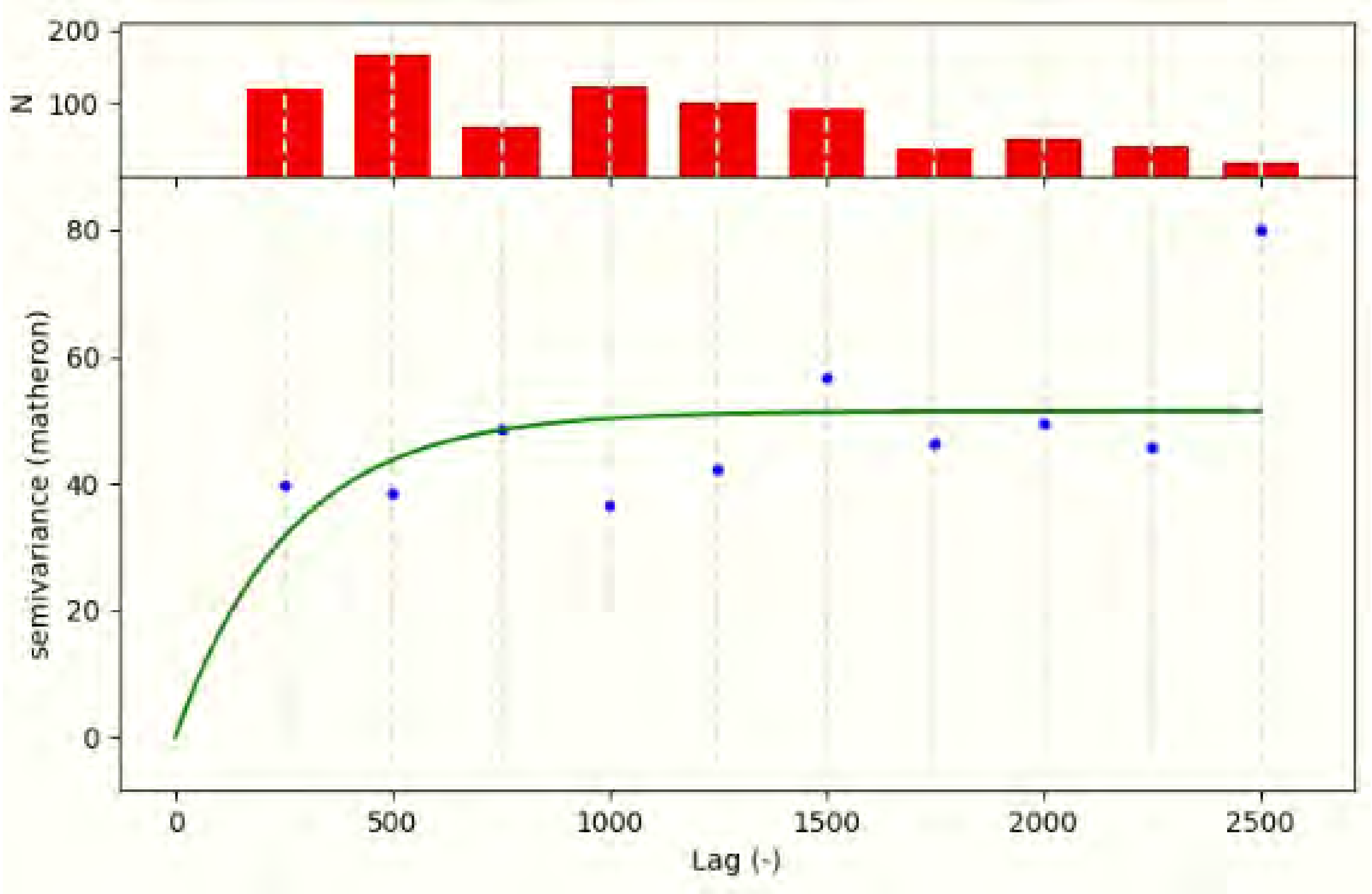


3

WP3 Task 3.3: Numerical simulation of alternative design cases & analysis of design methodologies

Spatial Characterization – Variogram Modelling

The variogram of the effective friction angle (**φ'**) was determined with the data from Soulou database:



$$\gamma(r) = \sigma^2 \cdot \left(1 - exp\left(\frac{r}{\theta}\right)\right) + n$$

Variogram Parameters:

- Exponential correlation function
- Variance = 51.4 (Statistical variance = 43.7)
- Correlation Length = 263.5m (Range = 790.7m)

Future Work:

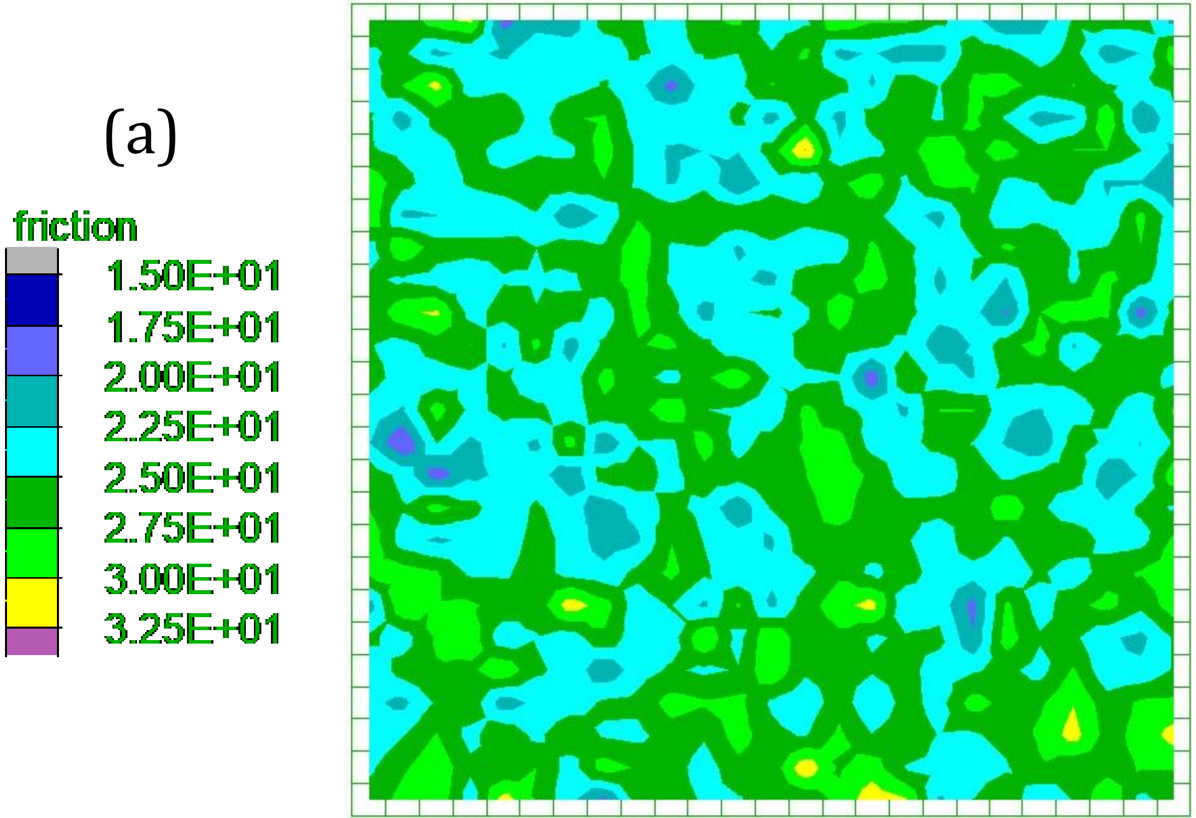
- Define the variograms of several geotechnical properties of Soulou spoil.
- Compare the resulting correlation lengths for the different properties to define possible patterns.
- Use the results to examine the effects of the spatially variable properties on SRF with FE analysis.

Spatial Random Field Generation - Examples

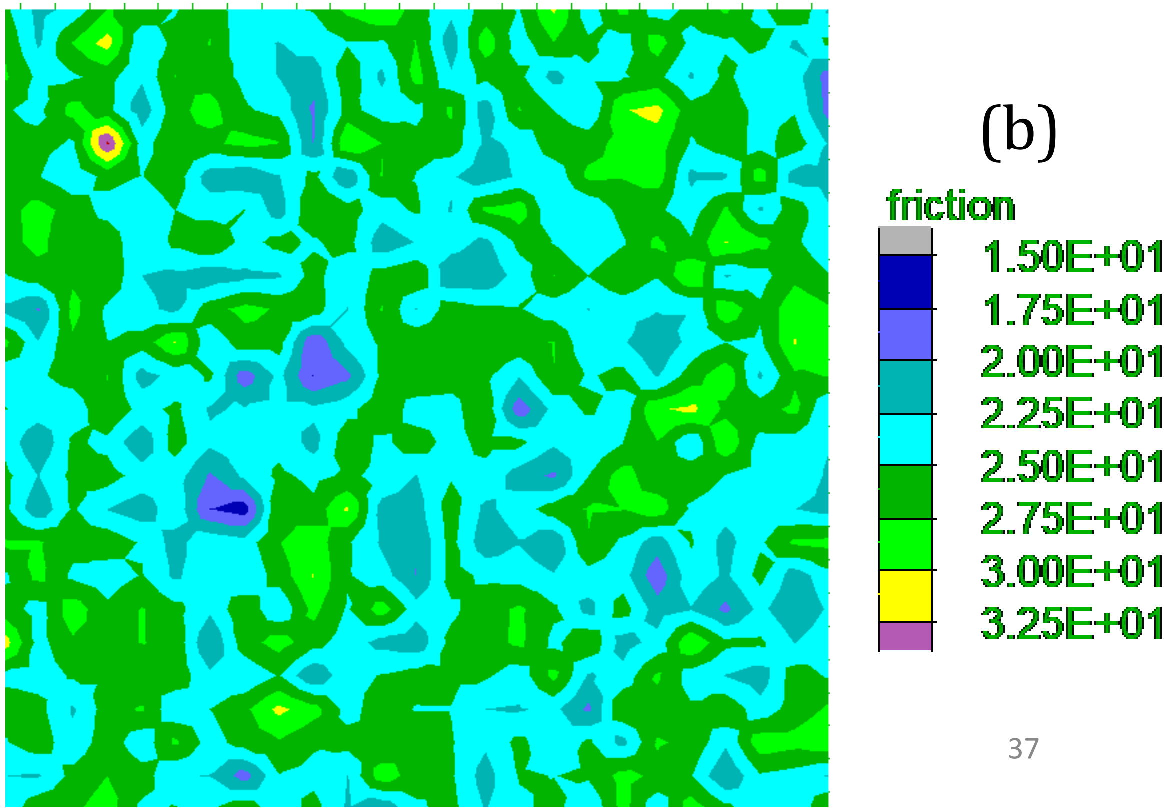
4 (random) realizations - named (a) to (d) – for the Spatial Random Field of the effective friction angle (**φ'**) are generated as an example to illustrate the effects of spatial variability on the problem of strip footing (Load-Displacement curve).

The variogram used is exponential and the correlation length is the same in all cases.

(a)

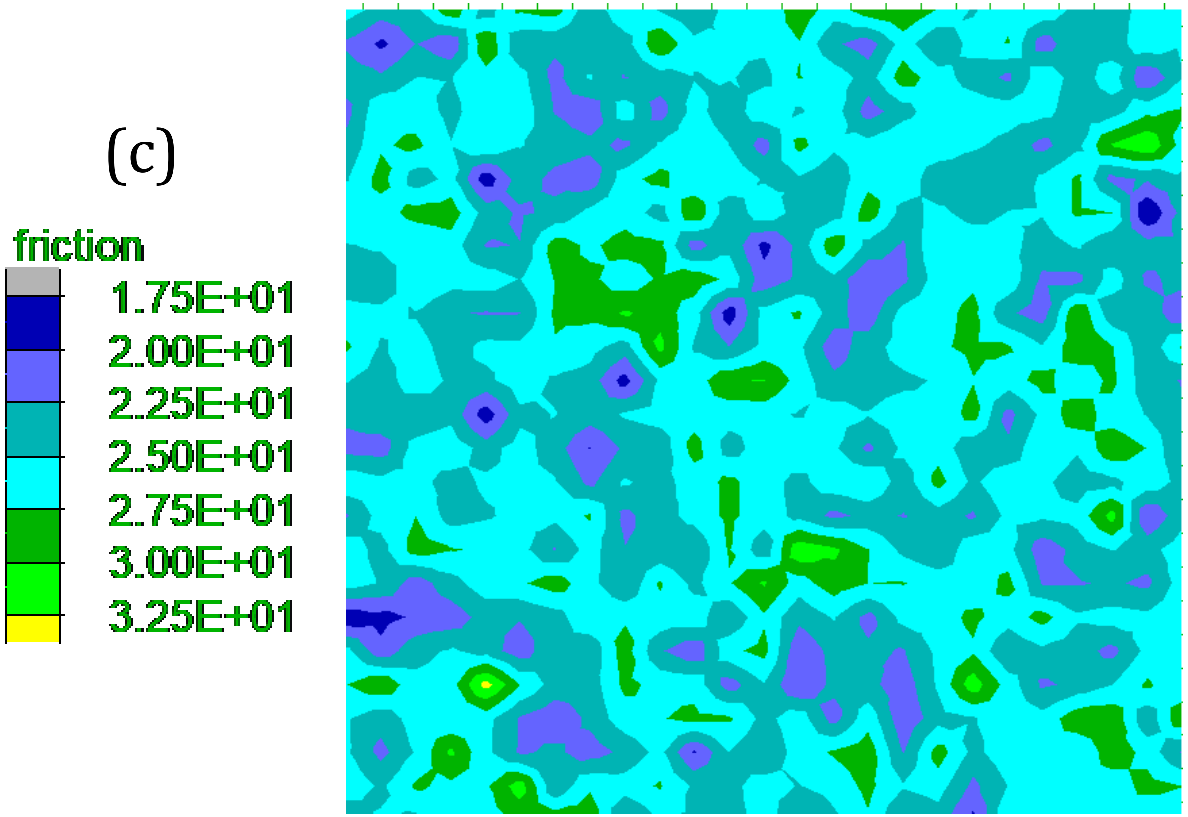


(b)

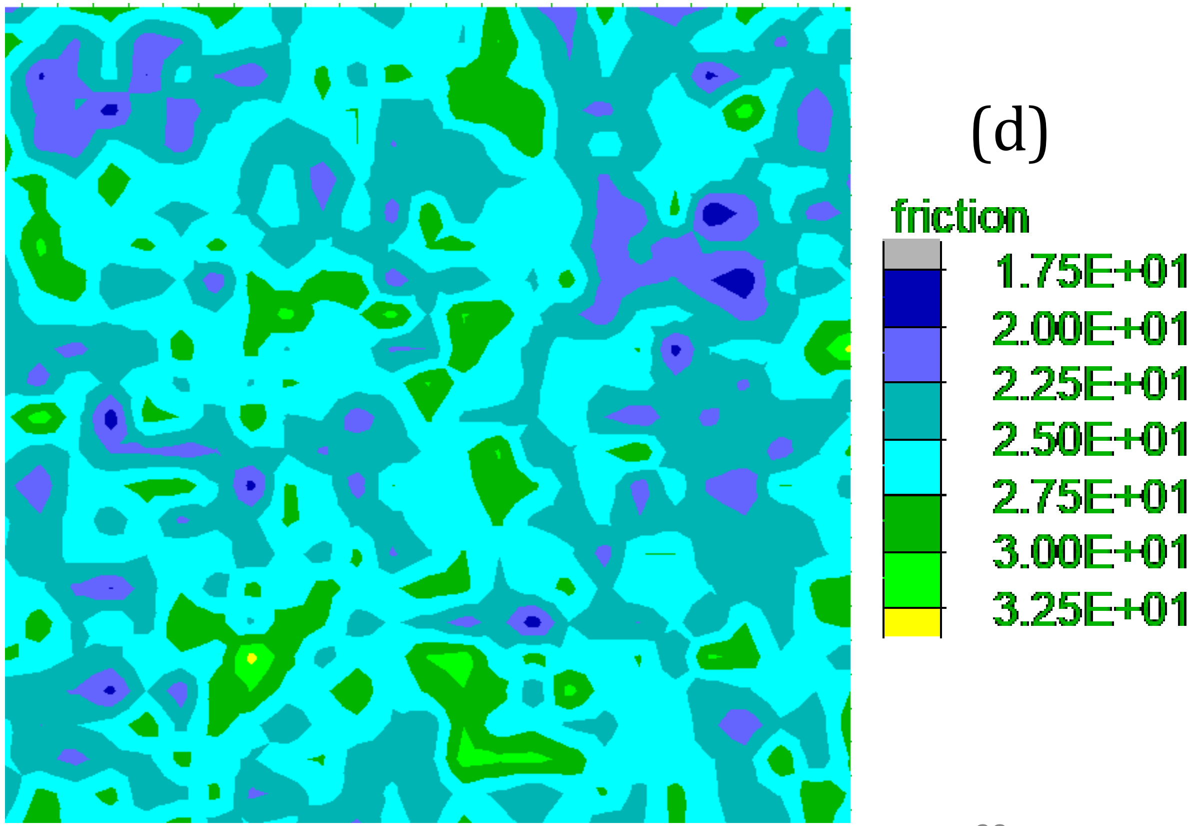


Spatial Random Field Generation - Examples

(c)

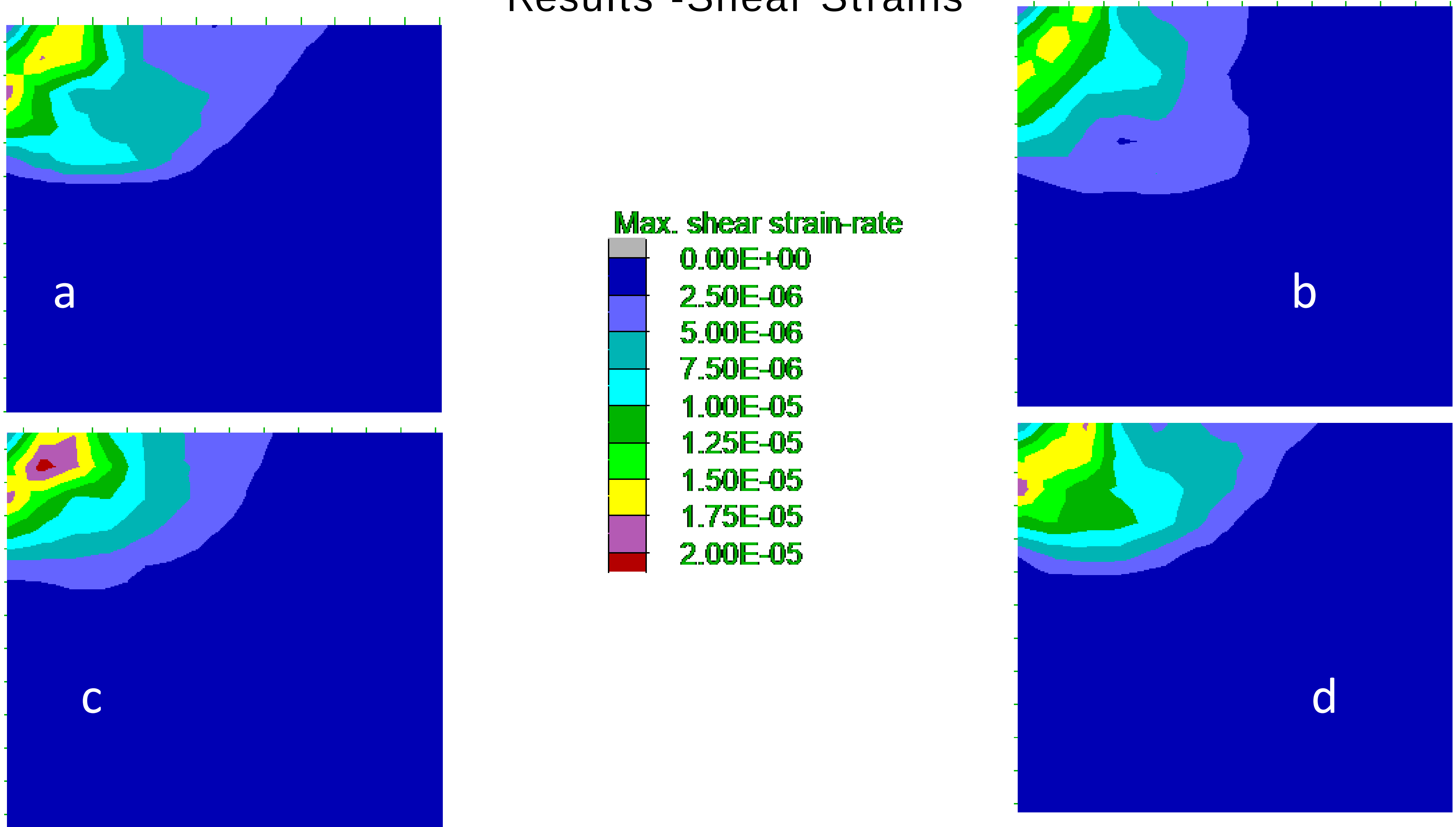


(d)



WP3 Task 3.3: Numerical simulation of alternative design cases & analysis of design methodologies

Spatial Random Fields (Examples) – Numerical Analysis in FLAC 2D Results -Shear Strains



Spatial Random Fields (Examples) – Numerical Analysis in FLAC 2D
Results: Load-Displacement curve